Solutions for Assignment #3

1. Solution of each part is as followed.

   (1) Time for loss of unconfined electrons = τ_e

$$\frac{1}{2}mv_e^2 = \frac{1}{2}KT_e$$

$$v_e = \sqrt{\frac{KT_e}{m}} = \sqrt{\frac{\left(1\times 10^3 eV\right)\left(1.602\times 10^{-19} J/eV\right)}{\left(9.11\times 10^{-31} kg\right)}} = 1.33\times 10^7 m/s$$

$$\tau_e = \frac{l}{v_e} = \frac{(1\,m)}{\left(1.33\times 10^7 m/s\right)} = 7.52\times 10^{-8}\ s$$

   (2) Time for loss of unconfined ions = τ_i

$$\frac{1}{2}mv_i^2 = \frac{1}{2}KT_i$$

$$v_i = \sqrt{\frac{KT_i}{m}} = \sqrt{\frac{\left(1\times 10^3 eV\right)\left(1.602\times 10^{-19} J/eV\right)}{\left(1.67\times 10^{-27} kg\right)}} = 3.1\times 10^5 m/s$$

$$\tau_i = \frac{l}{v_i} = \frac{(1\,m)}{\left(3.1\times 10^5 m/s\right)} = 3.2\times 10^{-6}\ s$$

   (3) Time for loss of many of the initially confined electrons

   The primary loss is due to small angle electron-electron collision, thus the time for loss = τ_{ee}

$$\nu_{ee} \sim 2.91\times 10^{-6}\frac{n_e \ln\Lambda}{T_e^{3/2}}$$

$$= 2.91\times 10^{-6}\frac{\left(10^{13} cm^{-3}\right)(20)}{(1000\ eV)^{3/2}}$$

$$= 1.84\times 10^4\ s^{-1}$$

$$\tau_{ee} = \frac{1}{\nu_{ee}} = \frac{1}{\left(1.84\times 10^4\ s^{-1}\right)} = 5.4\times 10^{-5}\ s$$

   Electric field will be induced due to the depletion of electrons. This field will retard the electron motion out of the mirror. Thus, not all of the electrons will leave.

   (4) Time for loss of many of the initially confined ions

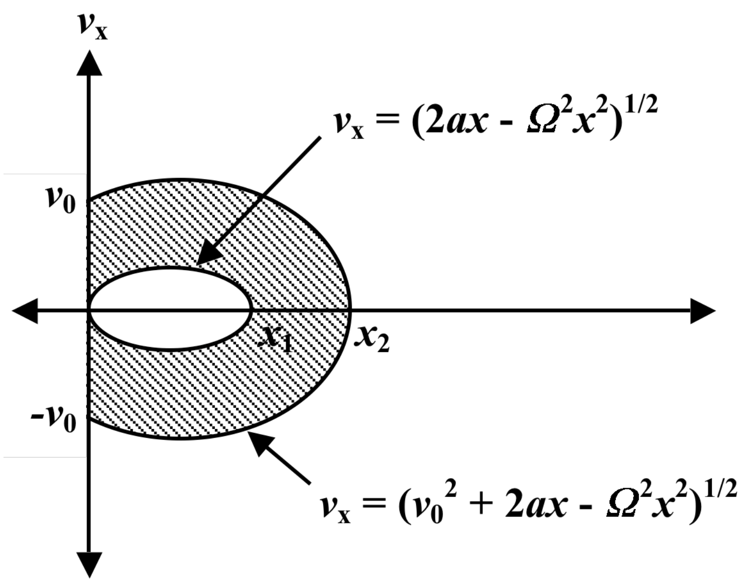
The primary loss is due to ion-ion collision, thus the time for loss = τ_{ii}

$$\nu_{ii} \sim \sqrt{\frac{m_e}{m_i}}\nu_{ee}$$
$$= 0.023\nu_{ee}$$
$$= 423\ s^{-1}$$
$$\tau_{ii} = \frac{1}{\nu_{ii}} = \frac{1}{\left(423\ s^{-1}\right)} = 2.4 \times 10^{-3}\ s$$

2. Total energy $\equiv \varepsilon = \frac{1}{2}m\left(v_x^2 + v_y^2\right) - eE_0x = \frac{1}{2}m\left(v_x^2 + \Omega^2x^2\right) - eE_0x$

$$\varepsilon\left(x=0\right) = \frac{1}{2}mu_0^2 = \varepsilon(\text{elsewhere})$$

Then, $u_0 = \sqrt{v_x^2 + \Omega^2x^2 - \frac{2eE_0x}{m}} \equiv \sqrt{v_x^2 + \Omega^2x^2 - 2ax}$, which is the constant of motion for this problem. The v_x-x plot is shown (only its shape) below.



Thus, for

$x < 0, \qquad n(x) = 0$

$0 < x < x_1, \qquad n(x) = \frac{2n_0}{v_0}\left\{\sqrt{v_0^2 + 2ax - \Omega^2x^2} - \sqrt{2ax - \Omega^2x^2}\right\}$

$x_1 < x < x_2, \qquad n(x) = \frac{2n_0}{v_0}\left\{\sqrt{v_0^2 + 2ax - \Omega^2x^2}\right\}$

$x_2 < x, \qquad n(x) = 0$

3. The first term in the first moment of Boltzmann equation can be written as

$$\frac{\partial}{\partial t}\left[mn\underline{u}\right] = mn\frac{\partial \underline{u}}{\partial t} + m\underline{u}\frac{\partial n}{\partial t} \tag{a}$$

The second term in the first moment of Boltzmann equation can be written as

$$\underline{\nabla}_x \bullet (nm\underline{u}\underline{u}) = (\underline{\nabla}_x \bullet \underline{u})mn\underline{u} + (\underline{u} \bullet \underline{\nabla}_x)mn\underline{u} \tag{b}$$

Note that (a) + (b) = $-\underline{\nabla}p + qn\left[\underline{E} + \underline{u} \times \underline{B}\right]$.

The zeroth moment of Boltzmann equation suggests that

$$\frac{\partial n}{\partial t} + \underline{\nabla}_x \bullet \left[n\underline{u}\right] = 0 \tag{c}$$

Multiplying (c) by $m\underline{u}$ from the left side and subtract from (a) + (b).

The left hand side of (a) + (b) – (d) is then $mn\frac{\partial \underline{u}}{\partial t} + mn(\underline{u} \bullet \underline{\nabla}_x)\underline{u}$.

The right hand side is $-\underline{\nabla}p + qn\left[\underline{E} + \underline{u} \times \underline{B}\right]$.

These are exactly the same as fluid equation.