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9.1 Introduction

Kinematics of rigid bodies involves both linear and angular quantities.

Usage
1. In designing the machines to perform the desired motion.
2. To determine the motion resulting from the applied force.

A rigid body A system of particles for which the distance between the particles remain unchanged. Thus there will be no change in the position vector of any particle measured from the body-fixed coordinate system.
### Ch. 9: Plane Kinematics of Rigid Bodies

#### 9.1 Introduction

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Plane motion of a rigid body: all parts of the body move in parallel planes. The body then can be treated as a thin slab with motion confined to the plane of motion; plane that contains the mass center.

Translation: motion in which every line in the body remains parallel to its original position at all time. That is, there is no rotation of any line in the body. The motion of the body is completely specified by the motion of any point in the body, since all points have the same motion.

Rectilinear translation: all points in the body move in parallel straight lines of the same distance.

Curvilinear translation: all points move on parallel curves of the same distance.

9.1 Introduction
Rotation motion in which all particles move in circular paths about the axis of rotation. All lines in the body which are perpendicular to the axis of rotation rotate through the same angle in the same time. Circular motion of a point helps describe the rotating motion.

General plane motion combination of translation and rotation. Principle of relative motion helps describe the general motion.

Approaches
1. Direct calculation of the absolute displacement, velocity, and acceleration from the geometry
2. Use the principle of relative motion
9.2 Rotation

Rotation of a rigid body is described by its angular motion, which is dictated by the change in the angular position (specified by angle $\theta$ measured from any fixed line) of any line attached to the body.

$$\theta_2 = \theta_1 + \beta$$
$$\Delta \theta_2 = \Delta \theta_1$$
$$\dot{\theta}_2 = \dot{\theta}_1$$
$$\ddot{\theta}_2 = \ddot{\theta}_1$$

All lines on a rigid body in its plane of motion have the same angular displacement, the same angular velocity, and the same angular acceleration.
Angular motion relations

Pick up any line in the plane of motion and associate it with the angular position coordinate $\theta$. The angular velocity $\omega$ and acceleration $\alpha$ of a rigid body in plane rotation are defined.

\[
\omega = \dot{\theta} \\
\alpha = \ddot{\omega} = \ddot{\theta} \\
\omega d\omega = \alpha d\theta \quad \text{or} \quad \dot{\theta} d\dot{\theta} = \ddot{\theta} d\theta
\]

Analogies between the linear and angular motion
Rotation about a fixed axis

All points other than those on the rotation axis move in concentric circles about the fixed axis. Curvilinear motion of a point A is related to the angular motion of the rigid body by the familiar $n$-$t$ coordinate kinematic relationship.

\[
v = r \omega, \; r = \text{constant}
\]

\[
a_t = \dot{v} = r \alpha
\]

\[
a_n = r \omega^2 = \frac{v^2}{r}
\]
Rotation about a *fixed axis*

\[ v = \dot{r} = \omega \times r \]  

by vector differentiation,  \[ |r| = \text{constant} \]

\( \omega = \text{angular velocity of the vector } r \)

\( = \text{angular velocity of the rigid body} \)

\[ a = \ddot{v} = \omega \times (\omega \times r) + \alpha \times r \]

\[ a_n = \omega \times (\omega \times r) \]

\[ a_t = \alpha \times r \]
The two V-belt pulleys form an integral unit and rotate about the fixed axis at O. At a certain instant, point A on the belt of the smaller pulley has a velocity \( v_A = 1.5 \text{ m/s} \), and point B on the belt of the larger pulley has an acceleration \( a_B = 45 \text{ m/s}^2 \) as shown. For this instant determine the magnitude of the acceleration \( a_C \) of point C and sketch the vector in your solution.
find $\omega_{pulley}$ from $v_A$

$[ v = \omega r ] \quad 1.5 = \omega \times 0.075, \quad \omega = 20 \text{ rad/s} \quad \text{CW}$

find $\alpha_{pulley}$ from $a_B$

$[ a_t = r \alpha ] \quad 45 = 0.4 \times \alpha, \quad \alpha = 112.5 \text{ rad/s}^2 \quad \text{CCW}$

acceleration at C

$[ a_n = r \omega^2 ] \quad a_{C_n} = 0.36 \times 20^2 = 144 \text{ m/s}^2$

$[ a_t = r \alpha ] \quad a_{C_t} = 0.36 \times 112.5 = 40.5 \text{ m/s}^2$

$a_C = \sqrt{a_{C_n}^2 + a_{C_t}^2} = 149.6 \text{ m/s}^2$
A V-belt speed-reduction drive is shown where pulley A drives the two integral pulleys B which in turn drive pulley C. If A starts from rest at time $t = 0$ and is given a constant angular acceleration $\alpha_1$, derive expressions for the angular velocity of C and the magnitude of the acceleration of a point P on the belt, both at time $t$. 

9.2 Rotation
9.2 Rotation

\[
\begin{align*}
\alpha &= \frac{d\omega}{dt} \\
\omega_A &= \alpha_1 t, \quad \omega_B = \left(\frac{r_1}{r_2}\right) \alpha_1 t, \quad \omega_C = \left(\frac{r_1}{r_2}\right)^2 \alpha_1 t
\end{align*}
\]

\[\alpha_A = \alpha_1, \quad \alpha_B = \left(\frac{r_1}{r_2}\right) \alpha_1, \quad \alpha_C = \left(\frac{r_1}{r_2}\right)^2 \alpha_1\]

\[
\begin{align*}
a_n &= r \omega^2 \\
a_p_n &= r_2 \left(\frac{r_1}{r_2}\right)^4 (\alpha_1 t)^2
\end{align*}
\]

\[
\begin{align*}
a_t &= r \alpha \\
a_p_t &= r_2 \left(\frac{r_1}{r_2}\right)^2 \alpha_1
\end{align*}
\]

\[
a_p = \sqrt{a_{p_n}^2 + a_{p_t}^2} = \frac{r_2^2}{r_1} \alpha_1 \sqrt{1 + \left(\frac{r_1}{r_2}\right)^4 \alpha_1^2 t^4}
\]
9.3 Absolute Motion

It is an approach to the kinematics analysis. It starts with the geometric relations that define the configuration involved. Then, the time derivatives of the relations are done to obtain velocities and accelerations. The +/- sense must be kept consistent throughout the analysis.

If the geometric configuration is too complex, resort to the principle of relative motion is recommended.
A wheel of radius $r$ rolls on a flat surface w/o slipping. Determine the angular motion of the wheel in terms of the linear motion of its center $O$. Also determine the acceleration of a point on the rim of the wheel as the point comes into contact with the surface on which the wheel rolls.
P. 9/3

For ROLLING w/o SLIPPING only:
displacement of the center $O = \text{arc length along the rim of the wheel}$
distance $s = \text{arc C'A} \rightarrow s = r\theta$
time derivatives: $v_0 = r\omega$ and $a_0 = r\alpha$
$v_0 = \text{velocity of the center } O$ of the wheel rolling w/o slipping
$\omega = \text{angular velocity of the wheel rolling w/o slipping}$
DO NOT mix up the rolling motion with the rotation about a fixed axis

Set up a fixed coordinate system originated at the point of contact between the rim and the ground.
Trajectory of a point $C$ on the rim is a cycloidal path.
$x$-$y$ coordinates of point $C$ at arbitrary position $C'$ is
$x = s - rsin\theta = r(\theta - \sin\theta) \quad y = r - r\cos\theta = r(1 - \cos\theta)$
time derivatives:
$\dot{x} = r\dot{\theta}(1 - \cos\theta) = v_0(1 - \cos\theta) \quad \dot{y} = r\dot{\theta}\sin\theta = v_0\sin\theta$
$\ddot{x} = \ddot{v}_0(1 - \cos\theta) + v_0\dot{\theta}\sin\theta = a_0(1 - \cos\theta) + r\omega^2\sin\theta$
$\ddot{y} = \ddot{v}_0\sin\theta + v_0\dot{\theta}\cos\theta = a_0\sin\theta + r\omega^2\cos\theta$
when point $C$ comes to contact, $\theta = 0$
$\dot{x} = 0, \dot{y} = 0, \ddot{x} = 0, \text{ and } \ddot{y} = r\omega^2$
P. 9/4  The load $L$ is being hoisted by the pulley and cable arrangement shown. Each cable is wrapped securely around its respective pulley so it does not slip. The two pulleys to which $L$ is attached are fastened together to form a single rigid body. Calculate the velocity and acceleration of the load $L$ and the corresponding angular velocity $\omega$ and angular acceleration $\alpha$ of the double pulley under the following conditions:

Case (a)  Pulley 1: $\omega_1 = 0$, $\alpha_1 = 0$  
(pulley at rest)  
Pulley 2: $\omega_2 = 2 \text{ rad/s}$  
$\alpha_2 = -3 \text{ rad/s}^2$

Case (b)  Pulley 1: $\omega_1 = 1 \text{ rad/s}$  
$\alpha_1 = 4 \text{ rad/s}^2$  
Pulley 2: $\omega_2 = 2 \text{ rad/s}$  
$\alpha_2 = -2 \text{ rad/s}^2$
Tangential displacement, velocity, and acceleration of a point on the rim of the wheel equal the corresponding motion of the points on the wrapped no-slip, inextensible cable.

(a) During time $dt$, line $AB$ (on the integral pulley) moves to $AB'$ through angle $d\theta$. New (tangential) position of point $A$ and $B$ are determined from the motion induced by the cable.

\[
\begin{align*}
    v_B &= v_D = r_2 \omega_2 = 0.2 \text{ m/s} \quad \text{and} \quad (a_B)_t = a_D = r_2 \alpha_2 = -0.3 \text{ m/s}^2 \\
    ds_A &= 0 \quad \therefore \text{left cable does not move} \\
    ds_B &= \overline{AB}d\theta \\
    v_B &= \overline{AB}\omega \\
    (a_B)_t &= \overline{AB}\alpha
\end{align*}
\]

$AB'$ maintains straight and point $O$ has absolute motion in vertical direction:

\[
\begin{align*}
    ds_O &= \overline{AO}d\theta \\
    v_O &= \overline{AO}\omega \\
    a_O &= \overline{AO}\alpha \\
    \therefore \omega &= 0.2 / 0.3 = 2 / 3 \text{ rad/s} \quad \text{CCW,} \quad \alpha = -0.3 / 0.3 = -1 \text{ rad/s}^2 \quad \text{CW} \\
    v_O &= 0.1 \times 2 / 3 = 0.0667 \text{ m/s} \quad \text{and} \quad a_O = 0.1 \times -1 = -0.1 \text{ m/s}^2
\end{align*}
\]
(b) During time $dt$, line $AB$ moves to $A'B'$ through angle $d\theta$.

New (tangential) position of point $A$ and $B$ are determined from the motion induced by the cable.

$v_A = v_C = r_1\omega_1 = 0.1 \text{ m/s}$ and $(a_A)_t = a_C = r_1\alpha_1 = 0.4 \text{ m/s}^2$

$v_B = v_D = r_2\omega_2 = 0.2 \text{ m/s}$ and $(a_B)_t = a_D = r_2\alpha_2 = -0.2 \text{ m/s}^2$

$ds_B - ds_A = \overline{AB}d\theta$  \hspace{1cm} $v_B - v_A = \overline{AB}\omega$  \hspace{1cm} $(a_B)_t - (a_A)_t = \overline{AB}\alpha$

$ds_O - ds_A = \overline{AO}d\theta$  \hspace{1cm} $v_O - v_A = \overline{AO}\omega$  \hspace{1cm} $a_O - (a_A)_t = \overline{AO}\alpha$

$\therefore \omega = (0.2 - 0.1)/0.3 = 1/3 \text{ rad/s \ CCW}$, $\alpha = (-0.2 - 0.4)/0.3 = -2 \text{ rad/s}^2 \text{ CW}$

$v_O = 0.1 + 0.1 \times 1/3 = 0.133 \text{ m/s}$ and $a_O = 0.4 + 0.1 \times -2 = 0.2 \text{ m/s}^2$
The telephone-cable reel rolls w/o slipping on the horizontal surface. If point A on the cable has a velocity $v_A = 0.8$ m/s to the right, compute the velocity of the center O and the angular velocity $\omega$ of the reel. (Be careful not to make the mistake of assuming that the reel rolls to the left.)
rolling w/o slipping: velocity at the contact point = 0
no slippage at the inner hub: velocity of the rim = velocity of the wrapped cable
OC remains straight and determines the angular motion
\[
\frac{v_o}{0.9} = \frac{0.8}{0.6}, \quad v_o = 1.2 \text{ m/s} \quad \rightarrow
\]

\[
[v_o = \omega r] \quad \omega = \frac{1.2}{0.9} = 1.333 \text{ rad/s} \quad \text{CW}
\]
The cable from drum A turns the double wheel B, which rolls on its hubs w/o slipping. Determine the angular velocity $\omega$ and angular acceleration $\alpha$ of drum C for the instant when the angular velocity and angular acceleration of A are 4 rad/s and 3 rad/s$^2$, respectively, both in the CCW direction.
[v = r\omega] \quad v_A = 0.2 \times 4 = 0.8 \, \text{m/s} \quad \rightarrow

[a_t = r\alpha] \quad (a_A)_t = 0.2 \times 3 = 0.6 \, \text{m/s}^2 \quad \rightarrow

\omega_B = \frac{0.8}{0.6} = \frac{4}{3} \, \text{rad/s} \quad \text{CW} \quad \alpha_B = \frac{0.6}{0.6} = 1 \, \text{rad/s}^2 \quad \text{CW}

v_C = \frac{4}{3} \times 0.2 = 0.267 \, \text{m/s} \quad \leftarrow

(a_C)_t = 1 \times 0.2 = 0.2 \, \text{m/s}^2 \quad \leftarrow

\omega_C = \frac{0.267}{0.2} = \frac{4}{3} \, \text{rad/s} \quad \text{CCW} \quad \alpha_C = \frac{0.2}{0.2} = 1 \, \text{rad/s}^2 \quad \text{CCW}

9.3 Absolute Motion
The rod OB slides through the collar pivoted to the rotating link at A. If OA has an angular velocity $\omega = 3$ rad/s for an interval of motion, calculate the angular velocity of OB when $\theta = 45^\circ$. 

![Diagram of the mechanism with labeled dimensions and angles]
given: $\theta = 45^\circ$, $\dot{\theta} = -3$ rad/s, $\overline{CA} = 0.2$ m/s

geometric relation:

$y\sin\beta - \overline{CA}\sin\theta = 0$ and $\overline{CA}\cos\theta + y\cos\beta = 0.4$

solve for $y = 0.2947$ m and $\beta = 28.675^\circ$

diff w.r.t. time:

$\dot{y}\sin\beta + y\dot{\beta}\cos\beta - \overline{CA}\dot{\theta}\cos\theta = 0$

$-\overline{CA}\dot{\theta}\sin\theta + \dot{y}\cos\beta - y\ddot{\beta}\sin\beta = 0$

$\therefore \dot{y} = -0.576$ m/s and $\dot{\beta} = -0.572$ rad/s
Show that the expressions $v = r\omega$ and $a_t = r\alpha$ hold for the motion of the center $O$ of the wheel which rolls on the concave or convex circular arc, where $\omega$ and $\alpha$ are the absolute angular velocity and acceleration, respectively, of the wheel. (Hint: Follow the sample problem and allow the wheel to roll a small distance. Be very careful to identify the correct absolute angle through which the wheel turns in each case in determining its angular velocity and angular acceleration.)
concave arc

geometry: $\theta = \text{angular motion of the wheel}$
rolling distance = $R \beta = r(\theta + \beta)$ \hfill (1)
motion of the center $O = s = (R - r) \beta$
rearrange (1) to give $(R - r) \beta = r \theta$
$v = \dot{s} = (R - r) \dot{\beta} = r \dot{\theta} = r \omega$
a$_t = \ddot{v} = (R - r) \ddot{\beta} = r \ddot{\omega} = r \alpha$

convex arc

geometry: $\theta = \text{angular motion of the wheel}$
rolling distance = $R \beta = r(\theta - \beta)$ \hfill (2)
motion of the center $O = s = (R + r) \beta$
rearrange (2) to give $(R + r) \beta = r \theta$
$v = \dot{s} = (R + r) \dot{\beta} = r \dot{\theta} = r \omega$
a$_t = \ddot{v} = (R + r) \ddot{\beta} = r \ddot{\omega} = r \alpha$

independent of the curvature, $R$, of the terrain!
The Geneva wheel is a mechanism for producing intermittent rotation. Pin P in the integral unit of wheel A and locking plate B engages the radial slots in wheel C thus turning wheel C one-fourth of a revolution for each revolution of the pin. At the engagement position shown, $\theta = 45^\circ$. For a constant CW angular velocity $\omega_1 = 2 \text{ rad/s}$ of wheel A, determine the corresponding CCW angular velocity $\omega_2$ of the wheel C for $\theta = 20^\circ$. (Note that the motion during engagement is governed by the geometry of triangle $O_1O_2P$ with changing $\theta$.)
triangle $O_1O_2P$:

$$
tan\beta = \frac{O_1P \sin \theta}{O_1O_2 - O_1P \cos \theta} = \frac{\frac{1}{\sqrt{2}} \sin \theta}{1 - \frac{1}{\sqrt{2}} \cos \theta}
$$

$$
\dot{\beta} \sec^2 \beta = \left(1 - \frac{1}{\sqrt{2}} \cos \theta\right) \times \frac{1}{\sqrt{2}} \dot{\theta} \cos \theta - \frac{1}{\sqrt{2}} \sin \theta \times \left(\frac{1}{\sqrt{2}} \dot{\theta} \sin \theta\right)
$$

given: $\theta = 20^\circ$, $\dot{\theta} = -2$ rad/s, $\ddot{\theta} = 0$

$\beta = 35.783^\circ$, $\dot{\beta} = -1.923$ rad/s

9.3 Absolute Motion
The rod AB slides through the pivoted collar as end A moves along the slot. If A starts from rest at $x = 0$ and moves to the right with a constant acceleration of $0.1 \text{ m/s}^2$, calculate the angular acceleration $\alpha$ of AB at the instant when $x = 150 \text{ mm}$. 
P. 9/10

\[ \ddot{x} = 0.1 \text{ m/s}^2 \text{ constant} \]
\[ \dot{x} = 0.1t \quad x = 0.05t^2 \]

geometry: \[ x = 0.2 \tan \theta \]
\[ \dot{x} = 0.2 \dot{\theta} \sec^2 \theta \]
\[ \ddot{x} = 0.2 \ddot{\theta} \sec^2 \theta + 0.2 \dot{\theta} \times \left( 2 \dot{\theta} \sec^2 \theta \tan \theta \right) \]

at \( x = 0.15 \text{ m} \), \( t = \sqrt{3} \text{ sec} \rightarrow \dot{x} = 0.1\sqrt{3} \text{ and } \ddot{x} = 0.1 \)

at that posture, \( \tan \theta = 3/4 \quad \sec \theta = 5/4 \)

\[ \therefore \dot{\theta} = 0.554 \text{ rad/s} \text{ and } \ddot{\theta} = -0.1408 \text{ rad/s}^2 \]
The punch is operated by a simple harmonic oscillation of the pivoted sector given by
\[ \theta = \theta_0 \sin 2\pi t \]
where the amplitude is
\[ \theta_0 = \pi / 12 \text{ rad (15°)} \]
and the time for one complete oscillation is 1 second. Determine the acceleration of the punch when (a) \( \theta = 0 \) and (b) \( \theta = \pi/12 \).
\[ \theta = \theta_0 \sin 2\pi t, \quad \dot{\theta} = 2\pi \theta_0 \cos 2\pi t, \quad \ddot{\theta} = -(2\pi)^2 \theta_0 \sin 2\pi t \]

\[ 0.1^2 = y^2 + 0.14^2 - 0.28y\cos \theta \]

\[ 0 = 2y\ddot{y} - 0.28\dot{y}\cos \theta + 0.28y \dot{\theta}\sin \theta \]

\[ 0 = 2\ddot{y}^2 + 2y\dddot{y} - 0.28\dot{y}\cos \theta + 0.28\ddot{y}\dot{\theta}\sin \theta \]

\[ + 0.28\dot{y}\dot{\theta}\sin \theta + 0.28y \dddot{\theta}\sin \theta + 0.28y \dot{\theta}^2 \cos \theta \]

when \( \theta = 0, \ t = 0, \ \dot{\theta} = 2\pi \theta_0, \ \ddot{\theta} = 0 \)

\[ y = 0.24, \ 0.04 \]

\[ \dot{y} = 0, \ \ddot{y} = -0.909 \text{ m/s}^2 \]

when \( \theta = \pi / 12, \ t = 1/4, \ \dot{\theta} = 0, \ \ddot{\theta} = -(2\pi)^2 \theta_0 \)

\[ y = 0.2284, \ 0.042 \]

\[ \dot{y} = 0, \ \ddot{y} = 0.918 \text{ m/s}^2 \]
One of the most common mechanisms is the slider-crank. Express the angular velocity \( \omega_{AB} \) and angular acceleration \( \alpha_{AB} \) of the connecting rod AB in terms of the crank angle \( \theta \) for a given constant crank speed \( \omega_O \). Take \( \omega_{AB} \) and \( \alpha_{AB} \) to be positive counterclockwise.
given:  \( \dot{\theta} = \omega_0, \quad \ddot{\theta} = 0 \)

geometry:  \( l \sin \beta = r \sin \theta \)

diff w.r.t. time:  \( l \dot{\beta} \cos \beta = r \dot{\theta} \cos \theta \)

\[
\dot{\beta} = \frac{r \omega_0 \cos \theta}{l \cos \beta} = \frac{r \omega_0}{l} \frac{\cos \theta}{\sqrt{1 - \frac{r^2}{l^2} \sin^2 \theta}}
\]

\[
l \ddot{\beta} \cos \beta - l \dot{\beta}^2 \sin \beta = r \ddot{\theta} \cos \theta - r \dot{\theta}^2 \sin \theta
\]

\[
\ddot{\beta} = \frac{l \ddot{\beta}^2 \sin \beta - r \dot{\theta}^2 \sin \theta}{l \cos \beta} = \frac{r \omega_0^2}{l} \sin \theta \frac{\frac{r^2}{l^2} - 1}{\left(1 - \frac{r^2}{l^2} \sin^2 \theta\right)^{3/2}}
\]
9.4 Relative Velocity

Principle of relative motion is another way to solve the kinematics problems. This method is usually suitable to the complex motion as it is more scalable.

Velocity propagation in the rigid body
Refer to Chapter 2, the relative velocity equation using the non-rotating reference frame is

\[ \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} \]

Let the two points, A and B, are on the same rigid body. The consequence of this choice is that the motion of one point as seen by an observer translating with the other point must be circular since the radial distance to the observed point from the reference point does not Changed.
Movement of the rigid body is partitioned into two parts: translation and rotation. In the figure, after the translation of the rigid body, expressed by the motion of B, the body appears to undergo fixed-axis rotation about B with A executing circular motion as shown in (b). Hence the relationship for circular motion describes the relative portion of A’s motion.

9.4 Relative Velocity
With B as the reference point, total displacement of A is
\[ \Delta \mathbf{r}_A = \Delta \mathbf{r}_B + \Delta \mathbf{r}_{A/B} \]
\[ \Delta \mathbf{r}_{A/B} = -\Delta \mathbf{r}_{B/A} \] has the magnitude \( r \Delta \theta \) as \( \Delta \theta \rightarrow 0 \)

**Relative linear motion** \( \Delta \mathbf{r}_{A/B} \) is accompanied by the **absolute angular motion** as seen from the translating axes x'-y'.

\[ \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} \]

If distance \( r \) between A and B is constant, \( \mathbf{v}_{A/B} \) is the velocity of the circular motion. That is

\[ \mathbf{v}_{A/B} = \omega \times \mathbf{r}_{A/B} \quad \mathbf{v}_{A/B} = r_{A/B} \omega \quad \mathbf{v}_{A/B} \perp \overline{AB} \]

\( \omega \) = absolute angular velocity of rigid body

9.4 Relative Velocity
9.4 Relative Velocity

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general motion = translational motion + rotational motion
Velocity propagation among rigid bodies
Refer to Chapter 7, the relative velocity equation using the non-rotating reference frame is

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

This time point A and B are coincident points on different rigid bodies for the instant. In this case, the distance between two points are not constrained to be fixed, even it is zero at the instant.

Point A is on the red linkage.
Point B is on the blue linkage.
They are, at the instant, coincided and are at the position of the pin in the slot.

9.4 Relative Velocity
Methods in solving the relative velocity equation

\[ \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B} \]

1. Vector algebra approach
2. Graphical analysis approach
3. Vector/Graphic approach

** Sketch of the vector polygon representing the vector equation is helpful!

Vector algebra approach
Write each term in terms of i- and j-components \( \rightarrow \) two scalar equations \( \rightarrow \) at most two unknowns.
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Graphical analysis approach
Known vectors are constructed using a convenient scale. Unknown vectors which complete the polygon are then measured directly from the drawing. This is suitable when the vector terms result in an awkward math expression.

Vector/Graphic approach
Scalar component equations may be written by projecting the vectors along convenient directions. Simultaneous equations may be avoided by a careful choice of the projections.
The wheel of radius $r = 300$ mm rolls to the right without slipping and has a velocity $v_o = 3$ m/s of its center O. Calculate the velocity of point A on the wheel for the instant represented.
\[ v_A = v_o + v_{A/O} \]
\[ v_{A/O} = \omega \times r_{A/O} \]
\[ \omega = v_o / r \]
\[ \omega = 3 / 0.3 = 10 \text{ rad/s CW} \]

\[ v_A = v_C + v_{A/C} \]
\[ v_C = 0 \] for rolling w/o slipping wheel
\[ v_{A/C} = \omega \times r_{A/C} \]
\[ v_A = 4.36 \text{ m/s } \angle 23.4^\circ \]
The power screw turns at a speed that gives the threaded collar C a velocity of 0.25 m/s vertically down. Determine the angular velocity of the slotted arm when $\theta = 30^\circ$. 

![Diagram of a power screw and slotted arm]
point A on the slotted arm, point B on the collar
A and B coincide at $\theta = 30^\circ$

Because of the sliding contact constraint from the slot, $v_{A/B}$ has the direction along the slot (away from O for CCW $\omega$).

$$\left[ v_A = v_B + v_{A/B} \right]$$

$$v_A = 0.25 \cos 30 = 0.217$$

$$\omega = \frac{v_A}{OA} = 0.417 \text{ rad/s \ CCW}$$

Imagine B immobile, A will be seen moving radially outward from O along the slot.
P. 9/15 The rotation of the gear is controlled by the horizontal motion of end A of the rack AB. If the piston rod has a constant velocity 300 mm/s during a short interval of motion, determine the angular velocity of the gear and the angular velocity of AB at the instant when x = 800 mm.
C is the point on the rack AB at where the line AB made a contact with the pitch of the gear. \(\therefore\) point C is constrained to have the same velocity as the coincident point on the gear (but not acceleration).

\[\therefore \mathbf{v}_C \perp \mathbf{OC}\]

\[\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}\]

\[\theta = \sin^{-1}(0.2/0.8) = 14.48^\circ\]

\[v_C = 0.3\cos\theta = \omega_0 \times 0.2\quad \omega_0 = 1.45\text{ rad/s} \quad \text{CW}\]

\[v_{C/A} = 0.3\sin\theta = \omega_{AB} \times \sqrt{0.8^2 - 0.2^2}\quad \omega_{AB} = 0.0968\text{ rad/s} \quad \text{CCW}\]
The flywheel turns CW with a constant speed of 600 rev/min, and the connecting rod AB slides through the pivoted collar at C. For the position of $\theta = 45^\circ$, determine the angular velocity of AB by using the relative velocity relations. (Choose a point D on AB coincident with C as a reference point whose direction of velocity is known.)
D = point on linkage AB that coincides with pivot point C on collar

\[
\begin{align*}
[v_D = v_C + v_{D/C}] & \quad v_D = v_{D/C} \text{ along the slot and pointing out of point A} \\
[v_A = v_D + v_{A/D}] & \quad v_A = 0.2 \times (600 \times 2\pi / 60) = 12.566 \text{ m/s} \\
\end{align*}
\]

\[v_{A/D} = v_A \sin 59.63 = 10.84 = \omega_{AB} \times 0.56\]

\[\omega_{AB} = 19.36 \text{ rad/s CW}\]
The Geneva mechanism is shown again here. By relative motion principles, determine the angular velocity of wheel C for $\theta = 20^\circ$. Wheel A has a constant CW angular velocity $\omega_1 = 2 \text{ rad/s}$.

9.4 Relative Velocity
P. 9/17

P is the point on wheel A at the knob
Q is the point on the wheel C, coincident with P when $\theta = 20^\circ$

\[
\left[ v_P = v_Q + v_{P/Q} \right] \quad v_{P/Q} \text{ along the slot pointing to } O_2
\]

\[
v_P = 2 \times \left( \frac{0.2}{\sqrt{2}} \right) = 0.283 \text{ m/s}
\]

\[
v_Q = v_P \sin 34.355 = \omega_2 \times 0.083 \quad \omega_2 = 1.924 \text{ rad/s CCW}
\]
At the instant represented, \( a = 150 \text{ mm} \) and \( b = 125 \text{ mm} \), and the distance \( a+b \) between \( A \) and \( C \) is decreasing at the rate of 0.2 m/s. Determine the common velocity \( v \) of points \( B \) and \( D \) for this instant.
\[
\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}
\]
\[
\mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C}
\]

constraint: \( \mathbf{v}_B = \mathbf{v}_D \)

\( \dot{a} + \dot{b} = -0.2 \) → block A and C become closer

and from the mechanism, A moves to the right and B to the left

\( \therefore \mathbf{v}_A - \mathbf{v}_C = 0.2 \mathbf{i} \text{ m/s} \)

from the velocity diagram

\[\frac{\mathbf{v}_B}{\tan 22.62} + \frac{\mathbf{v}_B}{\tan 36.87} = 0.2, \quad \mathbf{v}_B = 0.0536 \mathbf{j} \text{ m/s}\]
The wheel rolls w/o slipping. For the instant portrayed, when O is directly under point C, link OA has a velocity $v = 1.5$ m/s to the right and $\theta = 30^\circ$. Determine the angular velocity $\omega$ of the slotted link.

9.4 Relative Velocity
P. 9/19

\[ v_o = \omega r \]
\[ 1.5 = \omega_o \times 0.1, \quad \omega_o = 15 \text{ rad/s} \quad \text{CW} \]

\[ v_p = \omega_o \times (2 \times 0.1 \cos 15) = 2.9 \text{ m/s} \]

P is the point at the pin on the disk

Q is the coincident point on the slotted arm

\[ v_P = v_Q + v_{P/Q} \]

\( v_{P/Q} \) directs along the slot outward point C

\[ v_Q = v_p \cos (15 + 23.78) = 2.26 \text{ m/s} \]

\[ \omega_C = 2.26 / 0.124 = 18.23 \text{ rad/s} \quad \text{CCW} \]
Ends A and C of the connected links are controlled by the vertical motion of the piston rods of the hydraulic cylinders. For a short interval of motion, A has an upward velocity of 3 m/s, and C has a downward velocity of 2 m/s. Determine the velocity of B for the instant when $y = 150$ mm.
\[ \mathbf{v}_B = \mathbf{v}_C + \mathbf{v}_{B/C} = \mathbf{v}_A + \mathbf{v}_{B/A} \]

from the velocity diagram

\[ \frac{5}{\sin 60} = \frac{\mathbf{v}_{B/C}}{\sin 96.87} = \frac{\mathbf{v}_{B/A}}{\sin 23.13} \]

\[ \mathbf{v}_{B/C} = 5.732 \text{ m/s} \quad \text{and} \quad \mathbf{v}_{B/A} = 2.268 \text{ m/s} \]

\[ (\mathbf{v}_{B/C} \sin 23.13)^2 + (3 + \mathbf{v}_{B/A} \cos 83.13)^2 = \mathbf{v}_B^2 \]

\[ \mathbf{v}_B = 3.97 \text{ m/s} \quad \text{directed along the dotted arrow} \]
9.5 Instantaneous Center of Zero Velocity (ICZV)

Principle of relative motion find the velocity of a point on a rigid body by adding the relative velocity, due to rotation about a reference point, to the velocity of the reference point. If the reference point has zero velocity momentarily, the body may be considered to be in pure rotation about an axis, normal to the plane of motion, passing through this point. This point is called ICZV, which aids in visualizing and analyzing velocity in plane motion.

ICZV has zero velocity but \textit{not acceleration} \implies \text{not ICZA}
Point A and B, instantaneously, have **absolute circular motion** about point C
C = ICZV = instantaneous center of rotation
C may not be on the body physically, visualized as lying on the extended body
ICZV is not a fixed point in the body nor a fixed point in the plane
Ω = \( \frac{v_A}{r_A} = \frac{v_B}{r_B} \), \( v_B = \left( \frac{r_B}{r_A} \right) v_A \)
If the body translates only, ICZV is at infinity along the perpendicular line to the velocity
Vertical oscillation of the spring-loaded plunger F is controlled by a periodic change in pressure in the vertical hydraulic cylinder E. For the position $\theta = 60°$, determine the angular velocity of AD and the velocity of the roller A in its horizontal guide if the plunger F has a downward velocity of 2 m/s.
v_B is forced to move downward by the hydraulic
v_A is forced to move horizontally by the guide
∴ ICZV is at point C → v_D \perp CD
plunger moves down 2 m/s
∴ vertical component of v_D is 2 m/s downward

v_D = 2 / \cos 30 = \omega_{AD} \times (2 \times 0.1 \cos 30), \quad \omega_{AD} = 13.33 \text{ rad/s} \quad \text{CW}

v_A = \omega_{AD} \times (0.2 \sin 60) = 2.309 \text{ m/s}, \text{ to the right}
P. 9/22  
Determine the angular velocity $\omega$ of the ram head AE of the rock crusher in the position for which $\theta = 60^\circ$. The crank OB has an angular speed of 60 rev/min. When B is at the bottom of its circle, D and E are on a horizontal line through F, and lines BD and AE are vertical. The dimension are OB = 100 mm, BD = 750 mm, and AE = ED = DF = 375 mm. Carefully construct the configuration graphically, and use the method of Art. 5/5.
Home position of the machine
Graphical method is the best way for this problem!
Here we show the vector/graphic approach

from the initial and current posture,
between point O and F:

\[ 100 \sin 60 - 750 \cos \alpha + 375 \cos \beta = 375 \]
\[ -100 \cos 60 + 750 \sin \alpha + 375 \sin \beta = 850 \]
by the help of complex exponent, \( \alpha = 85.85^\circ, \beta = 23.9^\circ \)

between point O and A:

\[ 375 \cos \theta + 375 \cos \xi + 750 \cos \alpha - 100 \sin 60 = 375 \]
\[ -375 \sin \theta + 375 \sin \xi + 750 \sin \alpha - 100 \cos 60 = 475 \]
by the help of complex exponent, \( \theta = 81.04^\circ, \xi = 21.49^\circ \)
find distance from ICZV to point of interest

\[ l_1 \sin 30 = -100 \cos 60 + 750 \sin \alpha - l_2 \sin \beta \]

\[ l_1 \cos 30 = l_2 \cos \beta + 750 \cos \alpha - 100 \sin 60 \]

\[ l_1 = 773.63 \text{ mm}, \ l_2 = 768.2 \text{ mm} \]

\[ l_3 \cos \beta = l_4 \cos \theta + 375 \cos \xi \]

\[ l_4 \sin \beta = l_4 \sin \theta - 375 \sin \xi \]

\[ l_3 = 435.8 \text{ mm}, \ l_4 = 317.8 \text{ mm} \]

\( v_B = 60 \times 100 = (l_1 + 100) \times \omega_{BD}, \ \omega_{BD} = 6.868 \text{ rev/min CW} \)

\( v_D = \omega_{BD} \times l_2 = 5276 \text{ mm}\cdot\text{rev/min} = l_3 \times \omega_{DE}, \ \omega_{DE} = 12.1 \text{ rev/min CW} \)

\( v_E = \omega_{DE} \times l_4 = \omega_{AE} \times 375, \ \omega_{AE} = 10.26 \text{ rev/min} = 1.07 \text{ rad/s CW} \)
The shaft at O drives the arm OA at a clockwise speed of 90 rev/min about the fixed bearing at O. Use the method of ICZV to determine the rotational speed of gear B (gear teeth not shown) if (a) ring gear D is fixed and (b) ring gear D rotates CCW about O with a speed of 80 rev/min.
Motion of gear A is constrained by:
1. motion of link OA at point A: \( v_{A_{\text{gear}}} = v_{A_{\text{link}}} \)
2. motion of the leftmost point on the rim: \( v_{C_{\text{gearA}}} = v_{C_{\text{gearB}}} \)
3. motion of the rightmost point on the rim: \( v_{D_{\text{gearA}}} = v_{D_{\text{gearD}}} \)

(a) ring gear D is fixed
\( v_A = 90a \downarrow \)
\( v_c = 2 \times 90a = \omega_B \times a/2, \quad \omega_B = 360 \text{ rev/min} \quad \text{CW} \)

(b) ring gear D rotates 80 rev/min CCW
\( v_A = 90a \downarrow, \quad v_D = 80 \left( \frac{3a}{2} \right) = 120a \uparrow \)
\( v_c = \frac{10}{7} \left( 90a \right) = \omega_B \times a/2, \quad \omega_B = 600 \text{ rev/min} \quad \text{CW} \)
The large roller bearing rolls to the left on its outer race with a velocity of its center O of 0.9 m/s. At the same time the central shaft and inner race rotate CCW with an angular speed of 240 rev/min. Determine the angular velocity of each of the rollers.
v_o = 0.9 m/s →
ω_i = 240 rev/min = 8π rad/s CCW

\[ v = \omega r \]  ICZV of the inner race is \( \frac{0.9}{8\pi} \) m lower to point O

velocity of the point on the roller contacting w/ inner race

\[ = 8\pi \times \left( 0.05 - \frac{0.9}{8\pi} \right) = 0.3566 \text{ m/s} \rightarrow \]

ω_o = \( \frac{0.9}{0.125} \) = 7.2 rad/s CCW
velocity fo the point on the roller contacting w/ the outer race

\[ = 7.2 \times 0.025 = 0.18 \text{ m/s} \leftarrow \]

by similar triangle, \( \frac{0.18}{0.3566} = \frac{x}{0.05 - x} \), \( x = 16.77 \text{ mm} \)

ω_roller = \( \frac{0.18}{x} = 0.18 \) rad/s CW
9.6 Relative Acceleration

Relative acceleration relationship with nonrotating reference axes can be obtained from differentiating the relative velocity relation w.r.t. time:

\[ \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B} \]

\( \mathbf{a}_{A/B} \) is the acceleration that A appears to have to a nonrotating observer moving with B.

If A and B are two points on the same rigid body in plane motion, the distance between them remains constant so that the observer moving with B perceives A to have circular motion about B.
Therefore, relative acceleration term can be partitioned into the normal component, directed from A toward B due to the change of direction of $\mathbf{v}_{A/B}$, and the tangential component, perpendicular to AB due to change in magnitude of $\mathbf{v}_{A/B}$.

$$\mathbf{a}_{A} = \mathbf{a}_{B} + (\mathbf{a}_{A/B})_{n} + (\mathbf{a}_{A/B})_{t}$$

$$(\mathbf{a}_{A/B})_{n} = \omega \times (\omega \times \mathbf{r}), \quad (\mathbf{a}_{A/B})_{n} = r \omega^{2} = \frac{v_{A/B}^{2}}{r}$$

$$(\mathbf{a}_{A/B})_{t} = \alpha \times \mathbf{r}, \quad (\mathbf{a}_{A/B})_{t} = r \alpha = \dot{v}_{A/B}$$
9.6 Relative Acceleration
Methods in solving the relative acceleration equation

\[ \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B} \]

1. Vector algebra approach
2. Graphical analysis approach
3. Vector/Graphic approach

** Sketch of the vector polygon representing the vector equation is helpful!

Vector algebra approach
Write each term in terms of i- and j-components \(\rightarrow\) two scalar equations \(\rightarrow\) at most two unknowns.
Graphical analysis approach
Known vectors are constructed using a convenient scale. Unknown vectors which complete the polygon are then measured directly from the drawing. This is suitable when the vector terms result in an awkward math expression.

Vector/Graphic approach
Scalar component equations may be written by projecting the vectors along convenient directions. Simultaneous equations may be avoided by a careful choice of the projections.

Because $a_n$ depend on velocities, normally it is required to solve for the velocities before the acceleration calculation can be made.
If the wheel in each case rolls on the circular surface w/o slipping, determine the acceleration of the point C on the wheel momentarily in contact with the circular surface. The wheel has an angular velocity $\omega$ and an angular acceleration $\alpha$. 

9.6 Relative Acceleration
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\( v_c = 0 \) because no slipping

\[
[a_c = a_o + a_{c/o}] \quad \text{and} \quad v_o = r\omega i, \quad (a_o)_t = r\alpha i
\]

\[
(a) \quad a_c = (a_o)_t + (a_o)_n + (a_{c/o})_t + (a_{c/o})_n
\]

\[
= r\alpha i + \frac{r^2\omega^2}{R - r} j - r\alpha i + r\omega^2 j = r\omega^2 \left( \frac{R}{R - r} \right) j
\]

\[
|a_c| = r\omega^2 \left( \frac{R}{R - r} \right) > r\omega^2
\]

\[
(b) \quad a_c = (a_o)_t + (a_o)_n + (a_{c/o})_t + (a_{c/o})_n
\]

\[
= r\alpha i - \frac{r^2\omega^2}{R + r} j - r\alpha i + r\omega^2 j = r\omega^2 \left( \frac{R}{R + r} \right) j
\]

\[
|a_c| = r\omega^2 \left( \frac{R}{R + r} \right) < r\omega^2
\]
The simplified clam-shell bucket is shown. With the block at $O$ considered fixed and with the constant velocity of the control cable at $C$ equal to 0.5 m/s, determine the angular acceleration $\alpha$ of the right-hand bucket jaw when $\theta = 45^\circ$ as the bucket jaws are closing.

9.6 Relative Acceleration
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9.6 Relative Acceleration

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\[ (a_B)_t \]

\[ (a_{B/C})_t \]

\[ 0.2604 \]

\[ 0.0782 \]

\[ 50.345^\circ \]

\[ 574.24 \text{ mm} \]

\[ 600 \text{ mm} \]

\[ 62.155^\circ \]

\[ 50.345^\circ \]

\[ 67.5^\circ \]

\[ 500 \text{ mm} \]

\[ \theta \]

\[ ICZV \]

9.6 Relative Acceleration
velocity analysis  use ICZV

locate ICZV of the right jaw:  $\overline{CD} = \overline{CO} \tan 50.345 = 692.78$ mm

$\omega_{BC} = \frac{0.5}{\overline{CD}} = 0.7212 \text{ rad/s} \quad \text{CW closing}$

$v_B = \omega_{BC} \times (\overline{CO} / \cos 50.345 - \overline{BO}) = 0.2165 \text{ m/s}$

$\omega_{BO} = \frac{v_B}{\overline{BO}} = 0.361 \text{ rad/s} \quad \text{CCW}$

acceleration analysis

$[a_B = a_C + a_{B/C}] \quad (a_B)_n + (a_B)_t = (a_{B/C})_n + (a_{B/C})_t$

$(a_{B/C})_n = \overline{BC} \omega_{BC}^2 = 0.2604 \text{ m/s}^2 \quad (a_B)_n = \overline{BO} \omega_{BO}^2 = 0.0782 \text{ m/s}^2$

horizontal direction:

$0.2604 \cos 22.5 + (a_{B/C})_t \sin 22.5 - (a_B)_t \cos 50.345 - 0.0782 \sin 50.345 = 0$

vertical direction:

$0.2604 \sin 22.5 - (a_{B/C})_t \cos 22.5 - (a_B)_t \sin 50.345 + 0.0782 \cos 50.345 = 0$

$(a_{B/C})_t = -0.049 \text{ m/s}^2 \quad \text{(wrong direction)} \quad (a_B)_t = 0.2532 \text{ m/s}^2$

$\alpha_{BC} = \frac{0.049}{0.5} = 0.098 \text{ rad/s}^2 \quad \text{CW}$

9.6 Relative Acceleration
The mechanism where the flexible band F attached to the sector at E is given a constant velocity of 4 m/s as shown. For the instant when BD is perpendicular to OA, determine the angular acceleration of BD.
point E moves in circular path about O with velocity 4 m/s

\[ \omega_{OA} = \frac{4}{0.2} = 20 \text{ rad/s  CCW} \]

\[ v_A = \omega_{OA} \times 0.125 = 2.5 \text{ m/s} \uparrow \]

velocity analysis

\[ [v_D = v_A + v_{D/A}] \quad v_D / v_A = 3/4, \quad v_D = 1.875 \text{ m/s} \]

\[ v_{D/A} / v_A = 5/4, \quad v_{D/A} = 3.125 \text{ m/s} \]

\[ \omega_{BD} = v_D / 0.25 = 7.5 \text{ rad/s  CCW}, \quad \omega_{AD} = v_{D/A} / 0.25 = 12.5 \text{ rad/s  CCW} \]

acceleration analysis

\[ [a_D = a_A + a_{D/A}] \quad a_A = 0.125 \times 20^2 = 50 \text{ m/s}^2 \leftarrow \]

\[ (a_{D/A})_n = 0.25 \times 12.5^2 = 39.0625 \text{ m/s}^2 \]

\[ (a_D)_n = 0.25 \times 7.5^2 = 14.0625 \text{ m/s}^2 \uparrow \]

\[ (a_{D/A})_t \perp AD \quad \text{and} \quad (a_D)_t \perp BD \]

from the polygon,

\[ 39.0625 \times \frac{3}{5} - (a_{D/A})_t \times \frac{4}{5} - 14.0625 = 0, \quad (a_{D/A})_t = 11.72 \text{ m/s}^2 \]

\[ 39.0625 \times \frac{4}{5} + (a_{D/A})_t \times \frac{3}{5} + (a_D)_t - 50 = 0, \quad (a_D)_t = 11.72 \text{ m/s}^2 \]

\[ \alpha_{AD} = \frac{(a_{D/A})_t}{0.25} = 46.875 \text{ rad/s}^2 \text{  CCW}, \quad \alpha_{BD} = \frac{(a_D)_t}{0.25} = 46.875 \text{ rad/s}^2 \text{  CW} \]
Elements of the switching device are shown. If the velocity \( v \) of the control rod is 0.9 m/s and is slowing down at the rate of 6 m/s\(^2\) when \( \theta = 60^\circ \), determine the magnitude of the acceleration of C.
9.6 Relative Acceleration

acceleration diagram

\[
(a_{A/B})_n = 14.40 \text{ m/s}^2
\]

\[
a_B = 6 \text{ m/s}^2
\]
velocity analysis by ICZV of linkage ABC

\[ \omega_{AB} = \frac{0.9}{0.075 \cos 30} = 13.856 \text{ rad/s \ CCW} \]

acceleration analysis

\[ [a_A = a_B + a_{AB}] \quad \text{see the acceleration diagram} \]

vertical direction:

\[ 6 + 14.4 \sin 30 - (a_{A/B})_t \sin 60 = 0 \]

horizontal direction:

\[ a_A = 14.4 \cos 30 + (a_{A/B})_t \cos 60 \]

\[ (a_{A/B})_t = 15.242 \text{ m/s}^2 \quad a_A = 20.09 \text{ m/s}^2 \]

\[ \alpha_{AB} = \frac{(a_{A/B})_t}{AB} = 203.227 \text{ rad/s}^2 \ \text{CW} \]

\[ [a_C = a_B + a_{C/B}] \quad (a_{C/B})_t = 15.242 \text{ m/s}^2 \quad (a_{C/B})_n = 14.4 \text{ m/s}^2 \]

\[ a_C = (14.4 \cos 30 + 15.242 \cos 60) \mathbf{i} + (6 - 14.4 \sin 30 + 15.242 \sin 60) \mathbf{j} \]

\[ = 20.09 \mathbf{i} + 12.0 \mathbf{j}, \quad a_C = 23.4 \text{ m/s}^2 \]
An oil pumping rig is shown in the figure. The flexible pump rod D is fastened to the sector at E and is always vertical as it enters the fitting below D. The link AB causes the beam BCE to oscillate as the weighted crank OA revolves. If OA has a constant CW speed of 1 rev every 3 s, determine the acceleration of the pump rod D when the beam and the crank OA are both in the horizontal position shown.
9.6 Relative Acceleration
9.6 Relative Acceleration

acceleration diagram

\[
\begin{align*}
(a_B)_n &= 0.486 \text{ m/s}^2 \\
(a_A)_n &= 2.63 \text{ m/s}^2 \\
(a_B/A)_t &= 11.9^\circ \\
(\alpha_B/A)_t &= 16.7^\circ \\
\beta &= 78.1^\circ \\
\gamma &= 16.7^\circ
\end{align*}
\]

Solved by vector algebra or vector geometry gives
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\( \omega_{OA} = 1/3 \text{ rev/s} = 2\pi/3 \text{ rad/s} \) CW const

velocity analysis

\( v_A = \omega_{OA} \times OA = 0.4\pi \text{ m/s} \uparrow \)

find the ICZV of linkage AB (see the figure)

\( \omega_{AB} = v_A / 10.1 = 0.04\pi \text{ rad/s} \) CW

\( \omega_{CE} = \frac{v_B}{\sqrt{0.9^2 + 3^2}} = \frac{\omega_{AB} \times 9.92}{\sqrt{0.9^2 + 3^2}} = 0.398 \text{ rad/s} \) CW

acceleration analysis

\[ [a_B = a_A + a_{B/A}] \] see the acceleration polygon

horizontal direction:

\[ 2.632 - 0.046 \cos 78.1 - (a_{B/A})_t \cos 11.9 - (a_B)_t \sin 16.7 - 0.496 \cos 16.7 = 0 \]

vertical direction:

\[ -0.046 \sin 78.1 + (a_{B/A})_t \sin 11.9 - (a_B)_t \cos 16.7 + 0.496 \sin 16.7 = 0 \]

\( (a_{B/A})_t = 2.036 \text{ m/s}^2 \) \( (a_B)_t = 0.54 \text{ m/s}^2 \)

\( \alpha_{CE} = \frac{(a_B)_t}{\sqrt{0.9^2 + 3^2}} = 0.1724 \text{ rad/s}^2 \) CW

\( a_D = (a_E)_t = 3.3 \times \alpha_{CE} = 0.569 \text{ m/s}^2 \downarrow \)

9.6 Relative Acceleration
An intermittent-drive mechanism for perforated tape F consists of the link DAB driven by the crank OB. The trace of the motion of the finger at D is shown by the dotted line. Determine the acceleration of D at the instant shown when both OB and CA are horizontal if OB has a constant CW rotational velocity of 120 rev/min.
9.6 Relative Acceleration

acceleration diagram

\begin{align*}
q_B &= 7900 \text{ mm/s}^2 \\
(q_A)_n &= 3160 \text{ mm/s}^2
\end{align*}
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\[ \omega_{OB} = 120 \times \frac{2\pi}{60} = 4\pi \text{ rad/s} \text{ CW const} \]

from the velocity direction of point A and B, ICZV is at \( \infty \)

\[ \therefore \text{linkage AB translates downward with velocity} = \omega_{OB} \times \overrightarrow{OB} = 0.628 \text{ m/s} \]

\[ \omega_{AC} = \frac{\omega_{A}}{\overrightarrow{AC}} = 5.0265 \text{ rad/s} \text{ CW} \]

\[ [ \mathbf{a}_{A} = \mathbf{a}_{B} + \mathbf{a}_{A/B} ] \quad \mathbf{a}_{B} = \frac{\mathbf{V}_{B}^2}{\overrightarrow{OB}} = 7.896 \text{ m/s}^2 \]

\[ (\mathbf{a}_{A})_{n} = \frac{\mathbf{V}_{A}^2}{\overrightarrow{AC}} = 3.158 \text{ m/s}^2 \]

\[ (\mathbf{a}_{A/B})_{n} = 0 \quad \therefore \text{no rotation at this moment} \]

\[ (\mathbf{a}_{A/B})_{t} = (7.896 - 3.158) / \cos14.4775 = 4.893 \text{ m/s}^2 \]

\[ \alpha_{AB} = \frac{(\mathbf{a}_{A/B})_{t}}{\overrightarrow{AB}} = 24.467 \text{ rad/s}^2 \text{ CW} \]

\[ [ \mathbf{a}_{D} = \mathbf{a}_{B} + \mathbf{a}_{D/B} ] \quad (\mathbf{a}_{D/B})_{n} = 0 \quad \therefore \text{no rotation at this moment} \]

\[ (\mathbf{a}_{D/B})_{t} = \alpha_{AB} \times \overrightarrow{BD} = 7.34 \text{ m/s}^2 \]

\[ \mathbf{a}_{D} = (-7.896 + 7.34 \cos14.4775) \mathbf{i} + (7.34 \sin14.4775) \mathbf{j} \]

\[ = -0.789 \mathbf{i} + 1.835 \mathbf{j} \text{ m/s}^2 \quad a_{D} = 1.997 \text{ m/s}^2 \]
9.7 Motion Relative to Rotating Axes

So far, \( \mathbf{v}_{A/B} \) and \( \mathbf{a}_{A/B} \) are measured from nonrotating reference axes. However, there are many situations where motion is generated within or observed from a system that itself is rotating. In these cases, the solution is greatly facilitated by the use of rotating reference axes. An example is the motion of the fluid particle along the curved vane of a rotating pump.

Consider two particles A and B moving independently in plane motion. Motion of A is observed from a moving reference frame x-y that goes with B, and that rotates with an angular velocity \( \omega = \dot{\theta} \).
\[ \mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B} = \mathbf{r}_B + (x \mathbf{i} + y \mathbf{j}) \]

\( \mathbf{i} \) and \( \mathbf{j} \) are not constant since their directions change

\[ \frac{d\mathbf{i}}{dt} = \mathbf{\omega} \times \mathbf{i} = \mathbf{\omega} \mathbf{j} \quad \text{and} \quad \frac{d\mathbf{j}}{dt} = \mathbf{\omega} \times \mathbf{j} = -\mathbf{\omega} \mathbf{i} \]

\[ \therefore \dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \mathbf{\omega} \times \mathbf{r}_{A/B} + (\dot{x} \mathbf{i} + \dot{y} \mathbf{j}) \]

or \( \mathbf{v}_A = \mathbf{v}_B + \mathbf{\omega} \times \mathbf{r}_{A/B} + \mathbf{v}_{\text{rel}} \)
Relative Velocity

\[ \mathbf{v}_A = \mathbf{v}_B + \mathbf{\omega} \times \mathbf{r}_{A/B} + \mathbf{v}_{rel} \]

\( v_A \) = velocity of the particle A
\( v_B \) = velocity of the particle B
\( v_{A/B} \) = velocity of A relative to B
\( v_{rel} \) = velocity of A as seen from the observer

\[ \mathbf{v}_{rel} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} \]

\( v_{rel} \) = velocity of A as seen from an observer

*at anywhere* fixed to the rotating x-y axes

\[ \mathbf{\omega} \times \mathbf{r}_{A/B} = (\mathbf{v}_{rel} \text{ observed from nonrotating x-y}) - (\mathbf{v}_{rel} \text{ observed from rotating x-y}) \]

If we use nonrotating axes, there will be no \( \mathbf{\omega} \times \mathbf{r}_{A/B} \) term. This makes \( \mathbf{v}_{rel} = \mathbf{v}_{A/B} \), which means the velocity seen by the observer is the velocity of A relative to B.

If B coincides with A, \( \mathbf{r}_{A/B} = \mathbf{0} \). This makes \( \mathbf{v}_{rel} = \mathbf{v}_{A/B} \), which means the velocity seen by the observer is the velocity of A relative to B, even the observer is rotating.
Physically, there are two particles A and B, which may be on different rigid bodies. Imagine there is a rotating plate on which particle B is located. On this plate, imagine the virtual point P currently coincident with point A. Relationship between \( v_P \) and \( v_B \) is indicated by:

\[
v_P = v_B + \omega \times r_{P/B} = v_B + \omega \times r_{A/B}
\]

Since point A is on the different body, \( v_A \neq v_P \). The velocity of A as seen from P (actually from any point fixed to the rotating plate) is \( v_{rel} \) and is tangent to the path (slot) fixed in the rotating plate.
Visualization of Relative Velocity Equation

In other words, the slot is the trajectory seen by the observer fixed to the rotating plate. *It is not the absolute path of A* (which must be measured by the fixed observer).

The magnitude of \( v_{rel} \), or the relative speed, is \( \dot{s} \).

From the velocity of \( v_A \) and \( v_P \), it is concluded that \( v_{rel} = v_{A/P} \)

that is, the velocity of A seen by the rotating B or P is the same, and is equal to the velocity of A relative to P (not to B)

9.7 Motion Relative to Rotating Axes
Observers moving with different velocities (different $v_B$) on the same rotating x-y see the same $v_{rel}$.

Observers moving with same velocity, but are on different rotating x-y frames (different $\omega$), see different $v_{rel}$.

Nonrotating observer sees the resultant of circular motion plus relative velocity.

**9.7 Motion Relative to Rotating Axes**
Vector differentiation

Change of a vector w.r.t. time as seen from a general reference frame depends on the intrinsic change of the vector itself and the change of the vector induced by the motion of the reference frame, whether it be the translation or rotation. Here the interested frame is constrained not to translate, but rotate around its origin.

An arbitrary vector $\mathbf{V} = V_x \mathbf{i} + V_y \mathbf{j}$ has the time derivative

$$\left( \frac{d\mathbf{V}}{dt} \right)_{xy} = \left( \dot{V}_x \mathbf{i} + \dot{V}_y \mathbf{j} \right) + \left( V_x \mathbf{i} + V_y \mathbf{j} \right)$$

$$\left( \frac{d\mathbf{V}}{dt} \right)_{xy} = \left( \frac{d\mathbf{V}}{dt} \right)_{xy} + \omega \times \mathbf{V}$$

9.7 Motion Relative to Rotating Axes
Vector differentiation

This expression means the time derivative of V when measured in the fixed frame (total time derivative) is equal to the time derivative of V as measured in the rotating frame plus the compensation due to rotation of the reference frame.
Vector differentiation

More insights can be seen from the vector diagram. Vector $V$ changes in both direction and magnitude to $V'$. x-y changes by rotating with angular velocity $\omega$ while X-Y is fixed. During time $dt$, the observer in rotating x-y see the change in magnitude of $V$, $dV$, plus the change in direction, $Vd\beta$, due to relative rotation of $V$ to x-y. The change the observer recognized is $(dV)_{xy}$. What he did not notice is the rotation of $V$ induced by the rotation of x-y, $Vd\theta$. Imagine that $V$ is fixed to x-y. Hence its direction changes by the rotation of x-y, which is not known to the observer rotating together.
Relative Acceleration

Differentiating the relative velocity equation results in the relative acceleration equation:

\[ \mathbf{a}_A = \mathbf{a}_B + \dot{\mathbf{\omega}} \times \mathbf{r}_{A/B} + \mathbf{\omega} \times \dot{\mathbf{r}}_{A/B} + \dot{\mathbf{v}}_{\text{rel}} \]

Recall the vector differentiation relation,

\[ \dot{\mathbf{r}}_{A/B} = \mathbf{v}_{\text{rel}} + \mathbf{\omega} \times \mathbf{r}_{A/B} \]
\[ \dot{\mathbf{v}}_{\text{rel}} = \mathbf{a}_{\text{rel}} + \mathbf{\omega} \times \mathbf{v}_{\text{rel}} \]

\[ \dot{\mathbf{\omega}} = \left( \frac{d\mathbf{\omega}}{dt} \right)_{xy} + \mathbf{\omega} \times \mathbf{\omega}, \quad \therefore \left( \frac{d\mathbf{\omega}}{dt} \right)_{xy} = \left( \frac{d\mathbf{\omega}}{dt} \right)_{xy} \]

→ angular acceleration observed in fixed frame = that observed in the rotating frame

Note: \( \mathbf{r}_{A/B} = xi + yj \quad \mathbf{v}_{\text{rel}} = \dot{x}i + \dot{y}j \quad \mathbf{a}_{\text{rel}} = \ddot{x}i + \ddot{y}j \)

(\( \mathbf{v}_{\text{rel}} \) and \( \mathbf{a}_{\text{rel}} \) = velocity and acceleration seen by the rotating observer)

\[ \therefore \mathbf{a}_A = \mathbf{a}_B + \dot{\mathbf{\omega}} \times \mathbf{r}_{A/B} + \mathbf{\omega} \times \left( \mathbf{\omega} \times \mathbf{r}_{A/B} \right) + 2\mathbf{\omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}} \]
Imagine a plate rotating with the same rate as x-y frame. P is the point on the plate coincident with A. Therefore P is seen to have circular motion about nonrotating observer at B.

9.7 Motion Relative to Rotating Axes
Visualization of Relative Acceleration Equation

\[ \mathbf{a}_p = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r}_{A/B} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{A/B}) \]

\[ \mathbf{a}_A = \mathbf{a}_p + \mathbf{a}_{A/P}, \quad \therefore \mathbf{a}_{A/P} = 2\boldsymbol{\omega} \times \mathbf{v}_{rel} + \mathbf{a}_{rel} \neq \mathbf{a}_{rel} \text{ and } \neq \dot{\mathbf{v}}_{rel} \]

\( \mathbf{a}_{rel} = \) acceleration of \( A \) seen by the rotating observer as moving along the relative path (slot)

\( = \) rate of change of \( \mathbf{v}_{rel} \) as observed in the rotating frame

Treat the relative path as absolute, the direction of \( \mathbf{a}_{rel} \) will always point to the side where the center of curvature of the relative path is located.

The n-t coordinate system seems most appropriate.

\( \left( \mathbf{a}_{rel} \right)_t = \dot{s} \) tangent to the relative path

\( \left( \mathbf{a}_{rel} \right)_n = \frac{\mathbf{v}_{rel}^2}{\rho} \) toward the center of curvature of the relative path
Visualization of Relative Acceleration Equation

\[2\mathbf{\omega} \times \mathbf{v}_{\text{rel}} = \text{Coriolis acceleration}\]

= difference between the acceleration of A relative to P

as measured from nonrotating and from rotating axes

\[
\left( a_{A/P} \right)_{XY} = 2\mathbf{\omega} \times \mathbf{v}_{\text{rel}} + \left( a_{A/P} \right)_{xy} = 2\mathbf{\omega} \times \mathbf{v}_{\text{rel}} + a_{\text{rel}}
\]

Note: \( \left( v_{A/P} \right)_{XY} = \left( v_{A/P} \right)_{xy} = v_{\text{rel}} \)

\( \therefore v_A = v_P + v_{A/P} = v_P + \mathbf{\omega} \times 0 + v_{\text{rel}} = v_P + \left( v_{A/P} \right)_{xy} \)

\[
\left( a_{A/P} \right)_{XY} \neq \left( a_{A/P} \right)_{xy} = a_{\text{rel}} = \left( \dot{v}_{\text{rel}} \right)_{xy}
\]

\[
\left( \dot{v}_{A/P} \right)_{XY} \neq \left( a_{A/P} \right)_{XY} \quad \therefore \left( a_{A/P} \right)_{XY} = 2\mathbf{\omega} \times v_{\text{rel}} + a_{\text{rel}} \quad \text{but} \quad \left( \dot{v}_{A/P} \right)_{XY} = \dot{v}_{\text{rel}} = \mathbf{\omega} \times v_{\text{rel}} + a_{\text{rel}}
\]

\( a_{\text{rel}}, \dot{v}_{\text{rel}}, \) and \( a_{A/P} \) are all three different quantities

\( a_{\text{rel}}, \) the simplest to visualize, is the change of \( v_{\text{rel}} \) observed in rotating frame

\( \dot{v}_{\text{rel}}, \) one step more difficult, is the change of \( v_{\text{rel}} \) observed in fixed frame -- differ by \( \mathbf{\omega} \times v_{\text{rel}} \)

\( a_{A/P}, \) the most difficult to imagine, is the acceleration of A relative to P -- differ by \( 2\mathbf{\omega} \times v_{\text{rel}} \)
Visualization of Relative Acceleration Equation

One \( \omega \times \mathbf{v}_{\text{rel}} \) comes from change of \( \mathbf{v}_{\text{rel}} \) due to rotation of x-y.
The other \( \omega \times \mathbf{v}_{\text{rel}} \) is from change of \( \omega \times \mathbf{r}_{A/B} \) due to \( \left( \dot{\mathbf{r}}_{A/B} \right)_{xy} = \mathbf{v}_{\text{rel}} \)

\[
\mathbf{v}_{\text{rel}} = \mathbf{v}_{\text{rel}} - \omega \times \mathbf{r}_{A/B} = \mathbf{v}_{\text{rel}} - \omega \times \left( \mathbf{r}_{A/B} - \mathbf{r}_{A/B} \right) = \mathbf{v}_{\text{rel}} - \omega \times \mathbf{r}_{A/B}
\]

\[
\mathbf{a}_{\text{rel}} = \mathbf{a}_{\text{rel}} - \omega \times \left( \omega \times \mathbf{r}_{A/B} \right) + \frac{1}{2} \omega \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}
\]

\[
\mathbf{a}_A = \mathbf{a}_B + \dot{\omega} \times \mathbf{r}_{A/B} + \omega \times \left( \omega \times \mathbf{r}_{A/B} \right) + \frac{2}{2} \omega \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}
\]

\[
\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{P/B} + \mathbf{a}_{A/P}
\]

\[
\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{P}
\]

\[
\mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}
\]

Summary of Acceleration Equations

9.7 Motion Relative to Rotating Axes
P. 9/31 The crank OA revolves CW with a constant angular velocity of 10 rad/s within a limited arc of its motion. For the position $\theta = 30^\circ$, determine the angular velocity of the slotted link CB and the acceleration of A as measured relative to the slot in CB.
Ch. 9: Plane Kinematics of Rigid Bodies

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\[
\begin{align*}
[\mathbf{v}_A &= \mathbf{v}_P + \mathbf{v}_{A/P}] & \quad \mathbf{v}_A &= 0.2 \times 10 = 2 \text{ m/s} \\
\mathbf{v}_P &= 2 \cos 30 = 2 \times 0.2 \cos 30 \times \omega_{CB}, \quad \omega_{CB} = 5 \text{ rad/s \ CW} \\
\mathbf{v}_{rel} &= 2 \sin 30 = 1 \text{ m/s} \\
[\mathbf{a}_A &= \mathbf{a}_C + \omega_{CB} \times \omega_{CB} \times \mathbf{r}_{A/C} + \dot{\omega}_{CB} \times \mathbf{r}_{A/C} + 2 \omega_{CB} \times \mathbf{v}_{rel} + \mathbf{a}_{rel}] \\
\left|\omega_{CB} \times \omega_{CB} \times \mathbf{r}_{A/C}\right| &= 8.66, \quad \left|2 \omega_{CB} \times \mathbf{v}_{rel}\right| = 10, \quad a_{A} = \frac{v_{A}^2}{OA} = 20 \\
\text{from diagram, } &a_{rel} = 20 \cos 30 - 8.66 = 8.66 \text{ m/s}^2 \text{ along the slot towards } C \\
\left|\dot{\omega}_{CB} \times \mathbf{r}_{A/C}\right| &= 20 \cos 60 - 10 = 0, \quad \dot{\omega}_{CB} = 0
\end{align*}
\]

velocity diagram

9.7 Motion Relative to Rotating Axes
9.7 Motion Relative to Rotating Axes
Determine the angular acceleration $\alpha_2$ of wheel C for the instant when $\theta = 20^\circ$. Wheel A has a constant CW angular velocity of 2 rad/s.
geometry analysis:
\[
\tan \alpha = \frac{200 \sin 20}{\sqrt{2}}, \quad \alpha = 35.78 \quad \text{and} \quad \overrightarrow{PO_2} = 82.7 \text{ mm}
\]

velocity analysis:
\[
[v_A = v_p + v_{rel}] \quad v_A = 2 \times 0.2 / \sqrt{2} = 0.283 \text{ m/s}
\]
\[
v_p = v_A \cos 55.78 = 0.159 = \omega_2 \times 0.0827, \quad \omega_2 = 1.923 \text{ rad/s CCW}
\]
\[
v_{A/p} = v_A \sin 55.78 = 0.234 \text{ m/s}
\]

acceleration analysis:
\[
[a_A = \omega_2 \times \omega_2 \times r_{A/O} + \dot{\omega}_2 \times r_{A/O} + 2 \omega_2 \times v_{rel} + a_{rel}]
\]

consider in \( \dot{\omega}_2 \times r_{A/O} \) direction
\[
|\dot{\omega}_2 \times r_{A/O}| = 0.9 + 0.5656 \cos 34.22, \quad \omega_2 = 16.54 \text{ rad/s}^2 \text{ CCW}
\]
9.7 Motion Relative to Rotating Axes

Velocity diagram:
- $v_A$: Velocity of point A.
- $v_P$: Velocity of point P.
- $v_{A/P}$: Velocity of point A relative to P.

Acceleration diagram:
- $|2\omega_x v| = 0.9$
- $|\alpha_{xr}|$
- $a_A = 0.5656$
- $|\omega_x \omega_{xr}| = 0.306$
- $35.78^\circ$
- $34.22^\circ$
- $20^\circ$
The space shuttle A is in an equatorial circular orbit of 240-km altitude and is moving from west to east. Determine the velocity and acceleration which it appears to have to an observer B fixed to and rotating with the earth at the equator as the shuttle passes overhead. Use $R = 6378$ km for the radius of the earth. Also use Fig. 1/1 for the appropriate value of $g$ and carry out your calculations to 4-figure accuracy.
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\[ \mathbf{v}_A = \mathbf{v}_B + \omega_e \times \mathbf{r}_{A/B} + \mathbf{v}_{rel} \]

from appendix, \( \omega_e = 0.7292 \times 10^{-4} \) rad/s \( \therefore \mathbf{v}_B = \omega_e \times 6.378 \times 10^3 = 465.084 \) m/s ←
\( \mathbf{v}_A = 465.084 + 0.7292 \times 10^{-4} \times 2.4 \times 10^3 + \mathbf{v}_{rel} \)

normal acceleration of the shuttle is \( g \) towards the center of the earth

\[ (a_A)_n = g \left( \frac{R}{R + h} \right)^2 = \frac{v_A^2}{R + h}, \ g = 9.814 \text{ m/s}^2, \ \therefore v_A = 7766.79 \text{ m/s} \leftarrow \]

\( \therefore v_{rel} = 7284.205 \text{ m/s} = 26223 \text{ km/h} \leftarrow \)
\[ \left[ \mathbf{a}_A = \mathbf{a}_B + \omega_e \times \omega_e \times \mathbf{r}_{A/B} + \dot{\omega}_e \times \mathbf{r}_{A/B} + 2 \omega_e \times \mathbf{v}_{rel} + \mathbf{a}_{rel} \right] \]

assume the shuttle orbits with constant velocity in circular path \( \rightarrow \mathbf{a}_A = (a_A)_n \)

the observer's acceleration is governed by the rotation of the earth

\[ (a_B)_n = 6.378 \times 10^3 \times \omega_e^2 = 33.9 \times 10^{-3} \downarrow \quad (a_B)_t = 0 \quad \therefore \dot{\omega}_e = 0 \]

\[ 9.115 = 33.9 \times 10^{-3} + (0.7292 \times 10^{-4})^2 \times 6.378 \times 10^3 + 2 \times 0.7292 \times 10^{-4} \times 7284.205 + a_{rel} \]

\[ a_{rel} = 8.018 \text{ m/s}^2 \downarrow \]

9.7 Motion Relative to Rotating Axes
The figure shows the vanes of a centrifugal-pump impeller which turns with a constant CW speed of 200 rev/min. The fluid particles are observed to have an absolute velocity whose component in the r-direction is 3 m/s at discharge from the vane. Furthermore, the magnitude of the velocity of the particles measured relative to the vane is increasing at the rate of 24 m/s² just before they leave the vane. Determine the magnitude of the total acceleration of a fluid particle an instant before it leaves the impeller. The radius of curvature $\rho$ of the vane at its end is 8 inch.
\[ \mathbf{v}_A = \mathbf{v}_P + \mathbf{v}_{\text{rel}} \]

\[ \mathbf{v}_P = 200 \times \frac{2\pi}{60} \times 0.15 = \pi \text{ rad/s} \downarrow, \quad (\mathbf{v}_A)_r = 3 \text{ m/s} \]

\[ \mathbf{v}_{\text{rel}} = 3\sqrt{2} \text{ m/s} \quad \text{and} \quad (\mathbf{v}_A)_{\theta} = (\pi - 3) \text{ m/s} \]

\[ \mathbf{a}_A = \mathbf{a}_P + 2\mathbf{\omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}} \]

\[ \mathbf{a}_A = 0.15 \times \left( \frac{20\pi}{3} \right)^2 + 2 \times \frac{20\pi}{3} \times 3\sqrt{2} + 24 + \frac{(3\sqrt{2})^2}{0.2} \]

\[ = 13.187\mathbf{i} - 45.04\mathbf{j} \quad \text{m/s}^2 \]

\[ \mathbf{a}_A = 46.93 \text{ m/s}^2 \]

9.7 Motion Relative to Rotating Axes
The mechanism shown is a device to produce high torque in the shaft at O. The gear unit, pivoted at C, turns the right-handed screw at a constant speed \( N = 100 \) rev/min in the direction shown which advances the threaded collar at A along the screw toward C. Determine the time rate of change \( \dot{\omega}_{AO} \) of the angular velocity of AO as it passes the vertical position shown. The screw has 3 single threads per centimeter of length.

\[ \omega \]

403 mm

29.745

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9.7 Motion Relative to Rotating Axes
Rotation of the screw causes relative translation of collar A.

\[ 3 \text{ rev} = 1 \text{ cm} \rightarrow v_{A/P} = \frac{100}{3} \text{ cm/min} = 1/180 \text{ m/s} \]

\[ [v_A = v_p + v_{rel}] \]

\[ v_A = \frac{1/180}{\cos 29.745} = \omega_{OA} \times 0.2, \quad \omega_{OA} = 0.032 \text{ rad/s CCW} \]

\[ \frac{v_p}{v_{rel}} = \tan 29.745, \quad v_p = 0.0032 \text{ m/s}, \quad \omega_{PC} = \frac{v_p}{CP} = 0.00788 \text{ rad/s CCW} \]

\[ [a_A = a_p + 2\omega_{PC} \times v_{rel} + a_{rel}] \]

screw turns at constant rate \( a_{rel} = 0 \)

\[ (a_A)_n = 0.2 \times \omega_{OA}^2 = 204.8 \times 10^{-6} \text{ m/s}^2 \]

\[ (a_A)_n = \overrightarrow{AC} \times \omega_{PC}^2 = 25.41 \times 10^{-6} \text{ m/s}^2 \]

\[ |2\omega_{PC} \times v_{rel}| = 87.556 \times 10^{-6} \text{ m/s}^2 \]

in vertical direction:

\[ 204.8 \times 10^{-6} = 25.41 \times 10^{-6} \times \sin 29.745 + 87.556 \times 10^{-6} \times \cos 29.745 + (a_p)_t \cos 29.745 \]

\[ (a_p)_t = 133.8 \times 10^{-6} \text{ m/s}^2 \]

in horizontal direction:

\[ (a_A)_t = \dot{\omega}_{OA} \times 0.2 = -25.41 \times 10^{-6} \times \cos 29.745 + (87.556 + 133.8) \times 10^{-6} \times \sin 29.745 \]

\[ \dot{\omega}_{OA} = 438.8 \times 10^{-6} \text{ rad/s CW} \]
9.7 Motion Relative to Rotating Axes

\[(a_P)_n = 25.41E-6\]
\[
|2 \omega x v| = 87.556E-6
\]
\[(a_A)_n = 204.8E-6\]

\[(a_P)_t\]
\[(a_A)_t\]

acceleration diagram
P. 9/36  Determine the angular acceleration of link EC in the position shown, where $\omega = \dot{\beta} = 2 \text{ rad/s}$ and $\ddot{\beta} = 6 \text{ rad/s}^2$ when $\theta = \beta = 60^\circ$. Pin A is fixed to link EC. The circular slot in link DO has a radius of curvature of 150 mm. In the position shown, the tangent to the slot at the point of contact is parallel to AO.
P. 9/36

\[ \mathbf{v}_A = \mathbf{v}_P + \mathbf{v}_{rel} \]
\[ \mathbf{v}_P = 2 \times 0.15 = 0.3 \text{ m/s} \]

\[ \frac{\mathbf{v}_{rel}}{\mathbf{v}_P} = \tan 60, \quad \mathbf{v}_{rel} = 0.5196 \text{ m/s} \quad \text{and} \quad \mathbf{v}_A = \frac{\mathbf{v}_P}{\cos 60} = 0.6 \text{ m/s} \]

\[ \omega_{AC} = \frac{0.6}{0.15} = 4 \text{ rad/s} \quad \text{CW} \]

\[ \mathbf{a}_A = \mathbf{a}_P + 2\boldsymbol{\omega}_{OA} \times \mathbf{v}_{rel} + \mathbf{a}_{rel} \]

refer to the diagram

unknown: \( (a_A)_t \) and \( (a_{rel})_t \)

consider the direction normal to \( (a_{rel})_t \)

\( (a_A)_t \cos 60 + 2.4 \cos 30 + 0.9 = 2.0784 + 1.8 \)

\( (a_A)_t = 1.8 = 0.15 \alpha_{EC}, \quad \alpha_{EC} = 12 \text{ rad/s}^2 \quad \text{CCW} \)

velocity diagram

9.7 Motion Relative to Rotating Axes
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**9.7 Motion Relative to Rotating Axes**

- \[(a_A)_t\]
- \[(a_{rel})_t, 30°\]
- \[(a_{rel})_n = 1.8\]
- \[|2\omega x v_{rel}| = 2.0784\]
- \[(a_P)_n = 0.6\]
- \[(a_P)_t = 0.9\]

acceleration diagram