Unit Root Tests

Test for Stationarity

- Correlogram (Graphical)
- Unit Root Tests
  - –Dickey-Fuller
  - –Phillip-Perron

Dickey-Fuller's UR Test (1)

Assumptions

Y_t~AR(1)

Hypothesis

H_0: Y_t ~ non-stationary

H_1: Y_t ~ stationary

Dickey-Fuller's UR Test(2)

CASE I

Y_t~AR(1) w/o drift and trend

Testing Model

$$Y_t = \phi_1 Y_{t-1} + \varepsilon_t$$

Dickey-Fuller's UR Test(3)

Testing Model

$$Y_t - Y_{t-1} = (\phi_1 - 1)Y_{t-1} + \varepsilon_t$$

$$\Delta Y_t = \gamma_1 Y_{t-1} + \varepsilon_t$$

where $\gamma = \phi_1 - 1$

H_0: $\phi_1 \geq 1$ or $\gamma \geq 0$

H_1: $\phi_1 < 1$ or $\gamma < 0$ ← Necessary condition

Dickey-Fuller's UR Test(4)

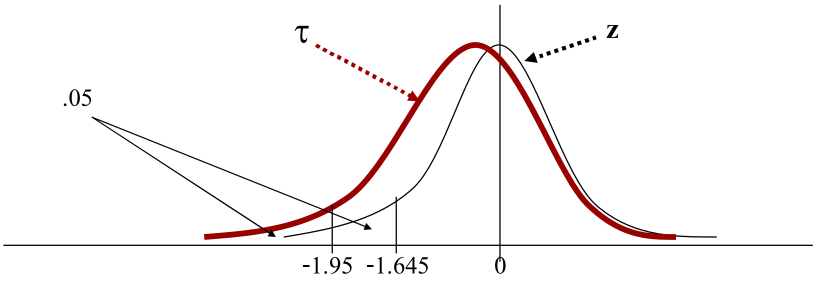
Steps

1) run OLS of ΔY_t on Y_{t-1}
2) test for $\gamma<0$

$$\tau = \frac{\hat{\gamma}}{se(\hat{\gamma})} \sim ??$$

DF have shown by simulation that τ does not follow a *t*-dist or a normal

Dickey-Fuller's UR Test(5)



Ref: Table 4.1 page 223 (Enders), Table D.7 (Gujarati)

Dickey-Fuller's UR Test(6)

<u>CASE II</u>

Y_t~AR(1) w/ drift but w/o trend

<u>Testing Model</u>

$$Y_t = a_0 + \phi_1 Y_{t-1} + \varepsilon_t$$

$$\Delta Y_t = a_0 + \gamma_1 Y_{t-1} + \varepsilon_t$$

Dickey-Fuller's UR Test(7)

Steps

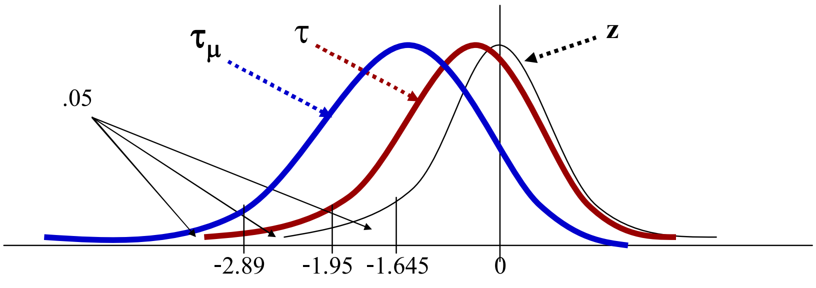
1) run OLS of ΔY_t on constant Y_{t-1}

2) test for $\gamma<0$

$$\tau_{\mu} = \frac{\hat{\gamma}}{se(\hat{\gamma})} \sim ??$$

DF have shown by simulation that τ_{μ} does not follow a *t*-dist or a normal

Dickey-Fuller's UR Test(8)



Ref: Table 4.1 page 223

Dickey-Fuller's UR Test(9)

CASE III

Y_t~AR(1) w/ drift and trend

Testing Model

$$Y_t = a_0 + \phi_1 Y_{t-1} + a_2 t + \varepsilon_t$$

$$\Delta Y_t = a_0 + \gamma_1 Y_{t-1} + a_2 t + \varepsilon_t$$

Dickey-Fuller's UR Test(10)

Steps

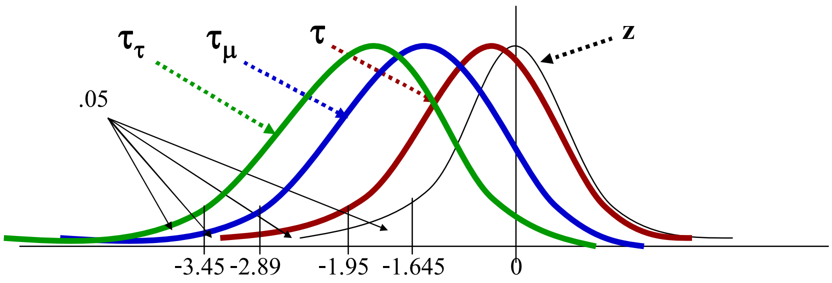
1) run OLS of ΔY_t on constant t Y_{t-1}

2) test for $\gamma<0$

$$\tau_\tau = \frac{\hat{\gamma}}{se(\hat{\gamma})} \sim ??$$

DF have shown by simulation that τ_τ does not follow a *t*-dist or a normal

Dickey-Fuller's UR Test(11)



Ref: Table 4.1 page 223

Dickey-Fuller's UR Test(12)

Note that

Drift and time trend will make it more difficult to reject H_0.

If H_0 is rejected when both are included, it will be rejected in the other cases.

Dickey-Fuller's UR Test(12)

Simple Rule

First, include both drift and trend in the testing model. Do τ_τ test.

If reject, Y ~ stationary

Otherwise, remove trend and do τ_μ test.

If reject, Y ~ stationary. Otherwise,

remove drift and do τ test.

If reject, Y ~ stationary. Otherwise, Y is non-stationary

Dickey-Fuller's UR Test(12)

Full-scheme testing rule

See Figure 4.7 on page 257

Augmented Dickey-Fuller (1)

Assumptions

$Y_t \sim AR(p)$

Testing Model

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_{p-1} Y_{t-p+1}$$
$$+ \phi_p Y_{t-p+1} - \phi_p (Y_{t-p+1} - Y_{t-p}) + \varepsilon_t$$

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Augmented Dickey-Fuller (2)

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + (\phi_{p-1} + \phi_p) Y_{t-p+2}$$
$$+ (-\phi_{p-1} - \phi_p)(Y_{t-p+2} - Y_{t-p+1})$$
$$+ (-\phi_p) \Delta Y_{t-p+1} + \varepsilon_t$$
$$= \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + (\phi_{p-1} + \phi_p) Y_{t-p+2}$$
$$+ (-\phi_{p-1} - \phi_p) \Delta Y_{t-p+2} + (-\phi_p) \Delta Y_{t-p+1} + \varepsilon_t$$
$$\vdots$$

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Augmented Dickey-Fuller (3)

$$\Delta Y_t = (\phi_1 + \phi_2 + \phi_2 + \cdots + \phi_p - 1)Y_{t-1}$$
$$+(-\phi_2 - \phi_2 - \cdots - \phi_p)\Delta Y_{t-1} + \cdots$$
$$+(-\phi_{p-2} - \phi_{p-1} - \phi_p)\Delta Y_{t-p+3}$$
$$+(-\phi_{p-1} - \phi_p)\Delta Y_{t-p+2}$$
$$+(-\phi_p)\Delta Y_{t-p+1} + \varepsilon_t$$

p-1 Augmented terms

Augmented Dickey-Fuller (4)

Hypothesis

H_0: $\gamma \geq 0$

H_1: $\gamma < 0$

where $\gamma = \phi_1 + \phi_2 + \phi_2 + \cdots + \phi_p - 1$

Fortunately, we can use the same $\boldsymbol{\tau_\tau}$, $\boldsymbol{\tau_\mu}$ and $\boldsymbol{\tau}$ tests as DF unit root tests

