Error Correction Model

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Covered Topics

- What is ECM = Error Correction Model or Mechanism?
- Restricted Auto-regressive model
- Estimation of ECM

Short and Long-term Models

The short-term model could be in the form of ARMAX

$$Y_t = \phi_0 + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \ldots + \phi_p Y_{t-p} + \beta_0 X_t + \beta_1 X_{t-1} + \ldots + \beta_K X_{t-K} + \varepsilon_t$$

Supposed that the long-term relationship follows

$$Y_t = \mu + \beta X_t + \varepsilon_t'$$

Error Correction Mechanism (1)

ECM is a mechanism for the dependent variable to adjust to its existing long-term trend. It explains how the long-term error will explain the movement in the short term.

Define ε_t' as the long-term error

In short term, change in Y will be explained by the long-term error

Error Correction Mechanism (2)

$$\begin{aligned}\Delta Y_t &= -\delta\varepsilon'_{t-1} \\ &\quad +\phi'_1\Delta Y_{t-1}+\phi'_2\Delta Y_{t-2}+\ldots+\phi'_{p-1}\Delta Y_{t-p+1} \\ &\quad +\beta'_0\Delta X_t+\beta'_1\Delta X_{t-1}+\ldots+\beta'_{K-1}\Delta X_{t-K+1}+\varepsilon_t \\ &= -\delta\,(Y_{t-1}-\mu-\beta\,X_{t-1}) \\ &\quad +\phi'_1\Delta Y_{t-1}+\phi'_2\Delta Y_{t-2}+\ldots+\phi'_{p-1}\Delta Y_{t-p+1} \\ &\quad +\beta'_0\Delta X_t+\beta'_1\Delta X_{t-1}+\ldots+\beta'_{K-1}\Delta X_{t-K+1}+\varepsilon_t\end{aligned}$$

Error Correction Mechanism (3)

There are p+K+1 coefficients in the restricted ARMAX model but there are p+K+3 parameters in the model

$$\begin{aligned}Y_t &= \delta\mu+(1-\delta+\phi'_1)Y_{t-1}+(\phi'_2-\phi'_1)Y_{t-2} \\ &\quad +\ldots+(\phi'_{p-1}-\phi'_{p-2})Y_{t-p+1}+(-\phi'_{p-1})Y_{t-p} \\ &\quad +\beta'_0X_t+(\beta'_1+\delta\beta)X_{t-1}+(\beta'_2-\beta'_1)X_{t-2} \\ &\quad +\ldots+(\beta'_{K-1}-\beta'_{K-2})X_{t-K+1}+(-\beta'_{K-1})X_{t-K}+\varepsilon_t\end{aligned}$$

Error Correction Mechanism (4)

Note that such an error correction mechanism is equivalent to a restricted short-term ARMAX model.

There could be more than one series of X

One of the X's could be time trend