2301610 Linear and Multilinear Algebra

EXERCISE 1.1

1. Prove that $\{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is continuous}\}$ is a subspace of $\mathbb{R}^{\mathbb{R}}$.

2. Let U and W be subspaces of a vector space V. Prove that $U \cup W$ is a subspace of V if and only if $U \subseteq W$ or $W \subseteq U$.

3. Let V be a vector space over a field and $W_1, W_2, \ldots, W_k \preceq V$. Prove that

(3.1) the sum $W_1 + W_2 + \cdots + W_k$ is a subspace of V which contains each of the subspace W_i;

(3.2) $W_1 + W_2 + \cdots + W_k = \langle W_1 \cup W_2 \cup \cdots \cup W_k \rangle$.

4. Let U and W be subspaces over a vector space V. Prove that

glb $\{U, W\}$ in $\{X \mid X \preceq V\} = U \cap W$ and lub $\{U, W\}$ in $\{X \mid X \preceq V\} = U \cup W = U + W$.

5. Let U be a subspace of a vector space V. For each $v \in V$, define

$$v + U := \{v + u \mid u \in U\}.$$

(5.1) Under what conditions is $v + U$ (where $v \in V$) a subspace of V? The set $v + U$ (where $v \in V$) is called an *affine subspace* of V.

(5.2) Show that any two affine subspaces of the form $v + U$ and $w + U$ are either equal or disjoint.

DUE TUESDAY 12 June 2007 before 13:00

Choose 3 problems from 5