2301610 Linear and Multilinear Algebra

EXERCISE 1.2

1. Let S be a basis for a vector space V. Prove that

(1.1) S is a maximal linearly independent subset of V;

(1.2) S is a minimal spanning subset of V.

2. In the proof of Theorem 1.8.13, show that the hypotheses of Zorn's Lemma are satisfied.

3. Let V be a finite-dimensional vector space and B a linearly independent subset of V. Prove that if $\operatorname{card} B = \dim V$, then B is a basis of V.

4. Let V be a finite-dimensional vector space and B span V. Prove that if $\operatorname{card} B = \dim V$, then B is a basis of V.

5. Let S be a subset of a vector space V such that S spans V. Prove that there exists a basis B for V such that $B \subseteq S$.

6. Show that there exist a vector space and its proper subspace with the same dimensions.

7. Let V be a vector space, $A \subseteq V$ which spans V, and C a linearly independent subset of A. Prove that there exists a basis B of V such that $C \subseteq B \subseteq A$.

DUE TUESDAY 26 June 2007 before 15:00

Hand in at least 4 problems:

Problems 5–7 are the must, choose 1 problem from Problems 1–4.