Game Theory Problem Set

3. *Betrand 2*: Consider the following Bertrand game with two firms: 1 and 2. The total market demand is Q = 10 − P. The total cost function of each firm is $C_1(q_1) = q_1^2$ and $C_2(q_2) = q_2^2$.

Then $\pi_i(p_1, p_2) = \pi^M(p_i)$ for $p_i < p_j$

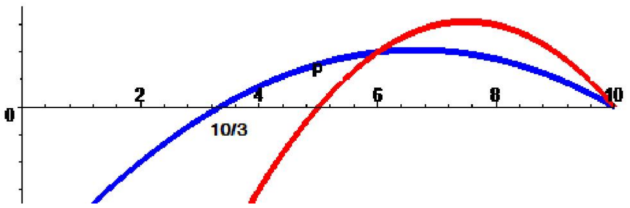
$\pi_i(p_1, p_2) = \pi^o(p_i)$ for $p_i = p_j$

$\pi_i(p_1, p_2) =$ 0 otherwise

where $\pi^M(p) = p(10-p) - (10-p)^2$, $\pi^O(p) = \dfrac{p_i(10-p)}{2} - (\dfrac{10-p}{2})^2$.

($\pi^M(p)$ is monopoly profit. $\pi^o(p)$ is a oligopoly profit.)

The graph of the two curves are shown below. The steeper curve is π^M. The other is π^o.



a) Draw $\pi_1(p_1, p_2)$ when $p_2 = 5.5$ and find $B_1(p_2)$ when $p_2 = 5.5$

b) Is $p_1 = p_2 = 5.5$ a NE?

c) Find all values of *p* such that $p_1 = p_2 = p$ is a NE.

4. *How to sell an exotic good*: Somsri has a unit of an exotic good to sell. Her objective is to maximize the expected revenue from selling. There are two potential buyers: Somchai and Somsak. The maximum value each potential buyer would pay for the good is *v*. We can say that *v* is the valuation of the good for Somchai and Somsak. They have the *same* valuation for the good. Somsri don't know the exact value of *v*. But Somsri knows that the probability that *v* = 10 or *v* = 40 is both a half.

a) Suppose Somsri sells the good by posting a price *p* and the customer who first pays the price *p* get the good. What is the optimal price that maximizes the expected revenue? What is her expected revenue? (For simplicity you can assume that one of the buyers will always buy the good if $p <= v$.)

b) Suppose Somsri uses a *discrete* first price auction and allows Somsak and Somchai to participate. In case that the two participants place the same bid, the winner is assigned randomly.

b1) Suppose *v* = 10. Find NEs. what is the minimum revenue that Somsri would get?

b2) Suppose *v* = 40. Find NEs. what is the minimum revenue that Somsri would get?

c) Which of the two selling mechanisms does give the higher expected revenue? In general, when auction is a good mechanism for selling good?

d) In question a) what would be the optimal price *p* if *v* is uniformly distributed in [0, 1]? (Assume that the price *p* is a real number.)

5. Consider a variant of the Stakelberg model with 3 firms. The market demand is Q = 1 – P. The cost of function each firm is zero. Consider the following scenario. The game has 2 periods. In period 1, firms 1 and 2 choose q1 and q2 simultaneously. Then in the second period, firm 3 chooses q3 after observing q1 and q2. Find the subgame perfect equilibrium *and* subgame perfect equilibrium outcome of the game.

6. *Coordination game*: Find all NEs (both pure and mixed) of the following game.

	L	M	R
L	1,1	0,0	0,0
M	0,0	1,1	0.0
R	0,0	0,0	1,1

7. (Monty Hall Problem) Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. (The host always opens the door with goat.) He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? Explain.

8. Consider the infinite-perod bargaining model that we studied in class where the discount factor of player 1 and player 2 are β_1 and β_2, respectively. This means that the value of the cake for player i = 1, 2 in period 1, 2 and 3 are S , $\beta_i S$, $\beta_i^2 S$. Find the SPE of the game.

9. Consider the following 2-period bargaining model. The model is similar to one we studied in class except that i) the game ends in period 2 and the discount factor of player 1 is 1 (player 1 is very patient), ii) the discount factor player 2 is β_2 iii) player 1 does not know the exact value of β_2 and believe that β_2 is uniformly distributed in [0,1]. Find the SPE of this game.

10. *Hitman*: A, B and C are fighting for a treasure box. Each man has a gun with a single bullet. Each person (if still alive) must shoot in his turn. First *A* can shoot, then *B* (if still alive) can shoot, then *C* (if still alive) can shoot. Let P_i for i = A,B,C is the probability that i hits the intended target he shoot at and $P_A = 1$, $P_B = 0.5$, and $P_C = x \in [0,1]$. If a person gets hit, he dies immediately. After all the 3 bullets are shot, the still alive persons share the treasure box equally. The objective of each man is to get as much as treasure as possible. i) Find the strategy of *A* in the subgame perfect equilibrium. (Hint: A has 3 possible options. You can assume that if a player is indifferent between two choices he chooses each choice with prob. ½. You may assume the utility from getting treasure box is 1 and utility from dying is 0.)

11. *Swapping Number*: Two players: 1 and 2 are playing the following game. Each player is randomly assigned an integer from 1 to 10 with equal probability. These numbers are private information for each player. Then each player simultaneously decides whether to swap his number with the other's number or not. If **both** decide to swap, the two numbers are swap. If the two numbers are swapped, *each* player gets 2.5 baht for swapping. Then the game ends and each player can exchange his number for money equal to his number. Find 2 NEs of this game.

12. Consider the first and second price auctions with 2 bidders we study in the class. Suppose the utility function of each bidder in case of win is

$$u_i(win) = (v_i - p)^\gamma \text{ if } v_i > p \quad \text{and } u_i(win) = v_i - p \text{ if } p_i \geq v$$

where p is his payment and γ is a positive number. In this situation, do the first and second price auctions still give the same revenue? (Your answer depend on γ.) (Assume that the valuation of each bidder is uniformly distributed in [0,1].)

13. Find the expected revenue of first and second price auction (with no reservation price) with n bidders' whose valuation is uniformly distributed in [0, 1].

14. *Discrete Second Price Auction with reservation price*: A seller sells on unit of good using a sealed-bid second price auction. There are two potential buyers. The valuation of each buyer has 2 values: 1(happens with probability *p*), 2(happen with probability 1-*p*). Each buyer knows only his valuation but does not know the valuation of the other buyer.

a) What would be the seller expected revenue under the usual second price auction?

b) Suppose the seller want to have highest expected revenue using the reservation price *R*. What is the optimal value of *R*? (Note: With a positive reservation price, each bidder can either submit a bid higher or equal to the reservation price or do not submit a bid. The winner must pay the second highest bid if there is a second bidder. If there is no second bidder, the winner must pay the reservation price.)

15. *All-pay auction*: Consider the following *independent* value all-pay auction in which each bidder must pay his bid no matter he wins or lose. Moreover, the bidder with the highest bid wins. Suppose there are 2 bidders and each bidder's valuation is uniformly distributed in [0, 1]. Find a NE of this game. (Hint: Guess that $b_i(v_i) = cv_i^d$ where *c*, *d* are some constant.)

16. *Asymmetric auction*: Consider the following *independent* value second-price auction with 2 asymmetric bidders. The first bidder's valuation is uniformly distributed in [0,1]. The first second's valuation is uniformly distributed in [0,2]. Find the seller's expected revenue.

17. A buyer and a seller are trading a good. The buyer's valuation for the good is Vb. The seller's valuation is Vs. Vs is the seller's private information and is uniformly distributed in [0,1]. The buyer does not know either Vb or Vs when trading. However, the buyer and the seller know that Vb = k.Vs and k > 1. Both buyer and seller know the value of *k*. In the first period, the buyer makes a single offer *p* in [0, 1]. In the second period, the seller decides to accept or reject the offer. (The utility of buyer and seller if offer is accepted Vb – p and p – Vs, respectively) Find the subgame perfect equilibriums when k < 2 and when k > 2.

19. Watch the 20-min video clip on http://www.ted.com/talks/lang/eng/laurie_santos.html and answer the following questions briefly based on the clip:

a) What is the similarity between human beings and monkeys? How does this similarity show about the source of mistakes human beings make?

b) What does the research implies about the assumption on utility maximization and expected utility commonly used in game theory?

c) Can human being learn from their mistakes and correct them? Yes/No/Uncertain discuss briefly.

d) (For fun.) If you are interested what else monkeys can do with money including prostitution, please check www.nytimes.com/2005/06/05/magazine/05FREAK.html

20. *Exponential Auction*: Find the expected revenue of standard first and second price auctions with 2 bidders. The valuation of each bidder is exponential distributed according to the following density function: $f_v(v) = e^{-v}$ for $v \geq 0$. Does the first or second price auction give higher expected revenue?

21. (Crime Reporting) Consider a variant of crime reporting game we consider in the class. Suppose the game is the similar to the one we see in the class except 1) there are only 3 people who see the crime scene and 2) to prevent false report, the police will come to save the victim if receiving at least to ***two*** calls. Find a symmetric Nash equilibrium of this game in case that v=2 and c=1.

22. (Defending territory) General A is defending territory accessible by two mountain passes against an attack by general B. General A has three divisions at her disposal, and general B has two divisions. Each general allocates her divisions between the two passes. General A wins the battle at a pass if and only if she assigns at least as many divisions to the pass as does general B; she successfully defends her territory if and only if she wins the battle at both passes. Is there any dominated strategy in this game? Find all mixed strategy equilibria in which no player use dominated strategy.

23. (Absent-Minded Driver) On his way to a party, John will pass 3 intersections. At each intersection, John has to decide whether to go straight or turn left. To go to the party, he needs to go straight at the first two intersections and turn left at the last intersection. If he misses a turn, he would get lost. However, John knows that he is very absent-minded and ***would never be able to distinguish*** the second and the last intersections. As a result of his absent-mindedness, he has to use the same strategy at the last two intersections. Suppose the payoffs from getting lost and joining the party are 0 and 1 respectively. Find his best strategy at the last 2 intersections. (Hint: you may need to consider a mixed strategy)

24. Consider an election game similar to the election game that we study in the lecture except that there are 3 candidates. Find a NE of this game. Show your logical argument in details.

25. Consider the game picking m stones with 3 players competing. Each player alternatively picks 1 or 2 stones in his turn. The player who picks the last stone is the first winner. The one who picks the stone just before the first winner is the second winner.
- Consider the case that $m = 6$. Which player will be the winner in this game?
- Consider the case that $m = 100$. Which player will be the winner in this game? Explain in details using backward induction.

26. Find 3 NEs of the Stackelberg game studied in class.

27. Consider the one-period bargaining game in which $u_i = x_i - \theta \mid x_1 - x_2 \mid$ where x_i is the amount of surplus player i has and $\theta > 0.$ is an inequality aversion parameter. Find the SPE of the two following cases. (Hint: i) for example if player 1 offer $\gamma = 0.25$ and player 2 accept, $u_i = 0.25s - \theta \mid 0.25s - 0.75s \mid$ ii) you answer depends on θ)

a) Player 2 is not allowed to reject the player1's offer.
b) Player 2 can either accept or reject the offer.