Homework 2: Due Wed 4 in TA class

1. A random variable X can take the values 1 and 2 with equal probability. For n equal to 2, demonstrate that $E(1/\bar{X})$ is not equal to $1/E(\bar{X})$.

2. Suppose we have the 4 following observation pairs of (X, Y).

X	Y
1	1
1	7
5	4
3	0

1.1 Find the values of a_0 and b_0 of the least-square line $\hat{Y}_i = b_0 + b_1 X_i$.

1.2 Find R^2 of the least square line.

1.3 Let $Z_i = \phi X_i + 1$ and ϕ is a constant. Find the values of a_0 and a_1 of the least-square line $\hat{Y}_i = a_0 + a_1 Z_i$.

3. Find the value of b that minimizes $\sum_{i=1}^{N} (Y_i - bX_i)^2$ in term of X_i and Y_i

4. Suppose we have only 2 observations: (X_1, Y_1) and (X_2, Y_2). If we find the least square line $\hat{Y}_i = b_0 + b_1 X_i$ using this two observations, what is the value of $\sum_i e_i^2$, we will get from the least-square line?

5. Given an example of estimator that is unbiased but not consistent.

6. A researcher has international data on aggregate wages, W, aggregate profits, P, and aggregate income, Y, for a sample of n countries. By definition, $Y_t = W_t + P_t$
The equations $\hat{W}_t = a_1 + a_2 Y_t$ $\hat{P}_t = b_1 + b_2 Y_t$ are estimated using least square estimator. Show that the regression coefficients will automatically satisfy the following equations: $a_2 + b_2 = 1$ and $a_1 + b_1 = 0$ Explain intuitively why this should be so.