Basic Concepts of Vibration

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**What is Vibration?**

**Vibration** is the study of the repetitive motion of objects relative to a stationary frame of reference or nominal position.
A component that stores and release potential energy is required.
1. Elastic components

Elastic components store or release potential (strain) energy as displacements increase or decrease.

e.g., helical spring, elastic bar & beam.

Restoring force

\[ f_k = -F = -kx \]

Potential energy

\[ V = \int_{0}^{x} kxdx = \frac{1}{2} kx^2 \]
Elementary parts of vibrating systems (2)

Combination of springs

Parallel

\[ k_{eq} = k_1 + k_2 + k_3 \]

Series

\[ k_{eq} = \left( \frac{1}{k_1} + \frac{1}{k_2} \right)^{-1} \]

\[ k_{eq} = \left( \sum_{i=1}^{n} \frac{1}{k_i} \right)^{-1} \]
Elementary parts of vibrating systems (3)

Elastic elements as springs

1. Thin rod

\[ k_{eq} = \frac{F}{\delta} = \frac{EA}{l} \]

2. Torsional bar

\[ k_{eq} = \frac{M}{\theta} = \frac{GJ_b}{l} \]
Elementary parts of vibrating systems (4)

Elastic elements as springs

3. Cantilever beam

\[ k_{eq} = \frac{F}{\delta} = \frac{3EI}{l^3} \]
2. Inertia (mass) components

Inertia components store or release kinetic energy as velocities increase or decrease.

e.g., mass (translation), mass moment of inertia (rotation)

Kinetic energy (translation)

\[ T = \frac{1}{2} m\dot{x}^2 \]

Kinetic energy (rotation)

\[ T = \frac{1}{2} J\dot{\theta}^2 \]
Vibration of the spring-mass system

**Ideal system**
There is no energy loss during vibration. The system will oscillate indefinitely.

**Real system**
Oscillating systems eventually die out and reduce to zero motion. There is a component that dissipates energy.
3. Viscous damper

Viscous damper or dashpot dissipates energy. Energy is converted to heat or sound.

\[ f_d = -F = -c\dot{x} \]

\[ k, m, c \text{ for rotational motion} \]

\[ F_k = kx \quad M_k = k_T\theta \]
\[ F_d = c\dot{x} \quad M_d = c_T\dot{\theta} \]
\[ F = m\ddot{x} \quad M = J\ddot{\theta} \]
Combination of springs (Example)

Find the equivalent single stiffness representation of the five-spring system shown in the figure.
Determine the torsional spring constant of the steel propeller shaft shown in the figure. (Shear modulus $G = 80$ GPa)

$k_{eq} = 6.6 \times 10^6$ N-m/rad
A hoisting drum, carrying a steel wire rope, is mounted at the end of a cantilever beam as shown in the figure. Determine the equivalent spring constant of the system when the suspended length of the wire rope is \( l \). Assume that the net cross-sectional diameter of the wire rope is \( d \) and the Young’s modulus of the beam and the wire rope is \( E \).

\[
k_{eq} = \frac{E}{4} \left( \frac{\pi at^3 d^2}{\pi d^2 b^3 + lat^3} \right)
\]