Design for Vibration Suppression

Outlines:

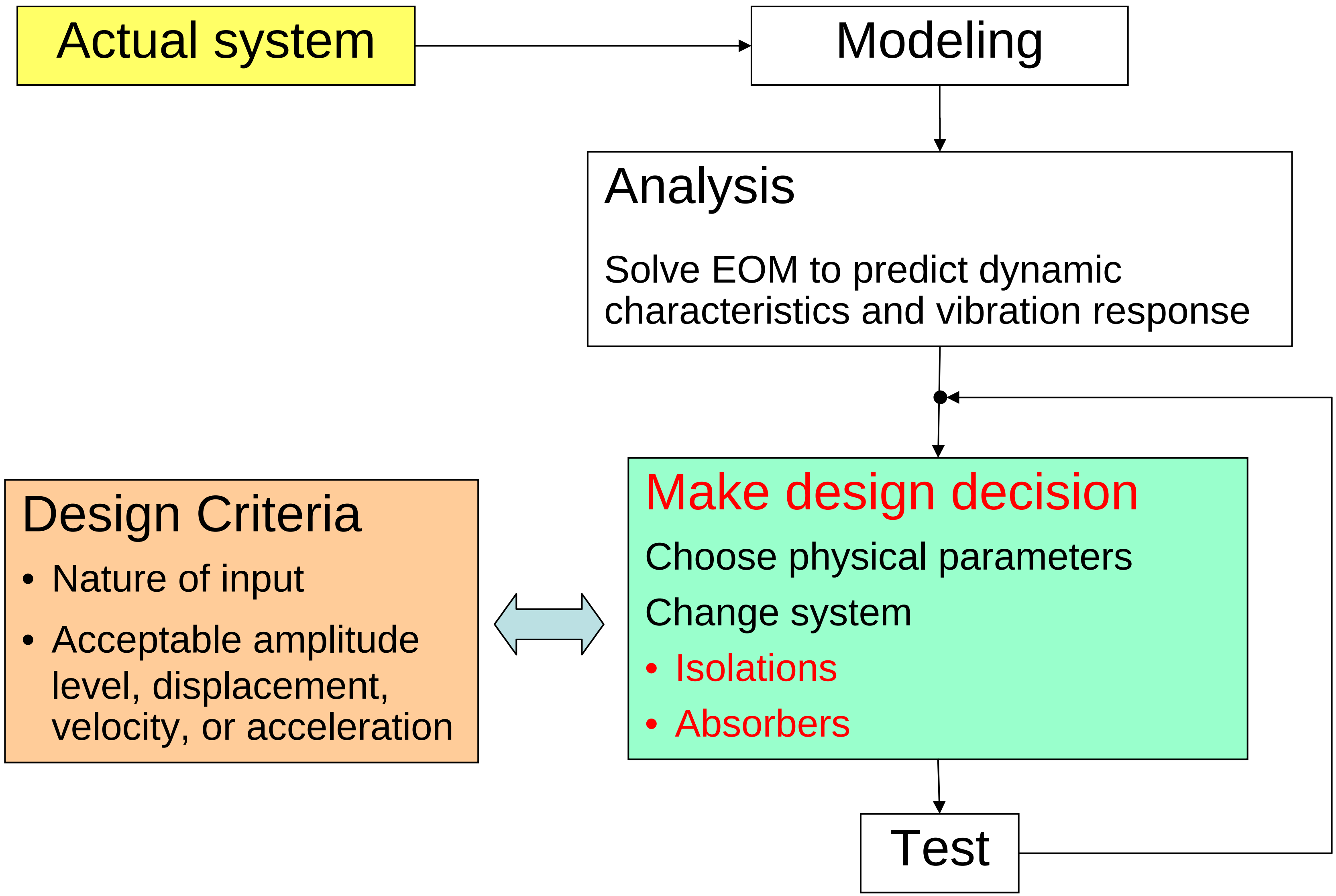
1. Vibration Design Process
2. Design of Vibration Isolation

   Moving base

   Fixed base

3. Design of Vibration Absorbers

Vibration Design Process



Nature of input / Design objectives

Nature of Input

1. **Vibration**:

   has some oscillatory features

2. **Shock**:

   sharp, aperiadic, and relatively short time

Design objectives

1. To protect the device from motion of its point of attachment (moving base)
2. To protect the point of attachment (ground) from vibration of the mass (isolate vibration from the source)

Isolation

3. To reduce vibration of the mass ⟹ Absorber

Acceptable level of vibration (1)

Level of vibration: displacement, velocity or acceleration

Standards are tested in terms of **rms** values

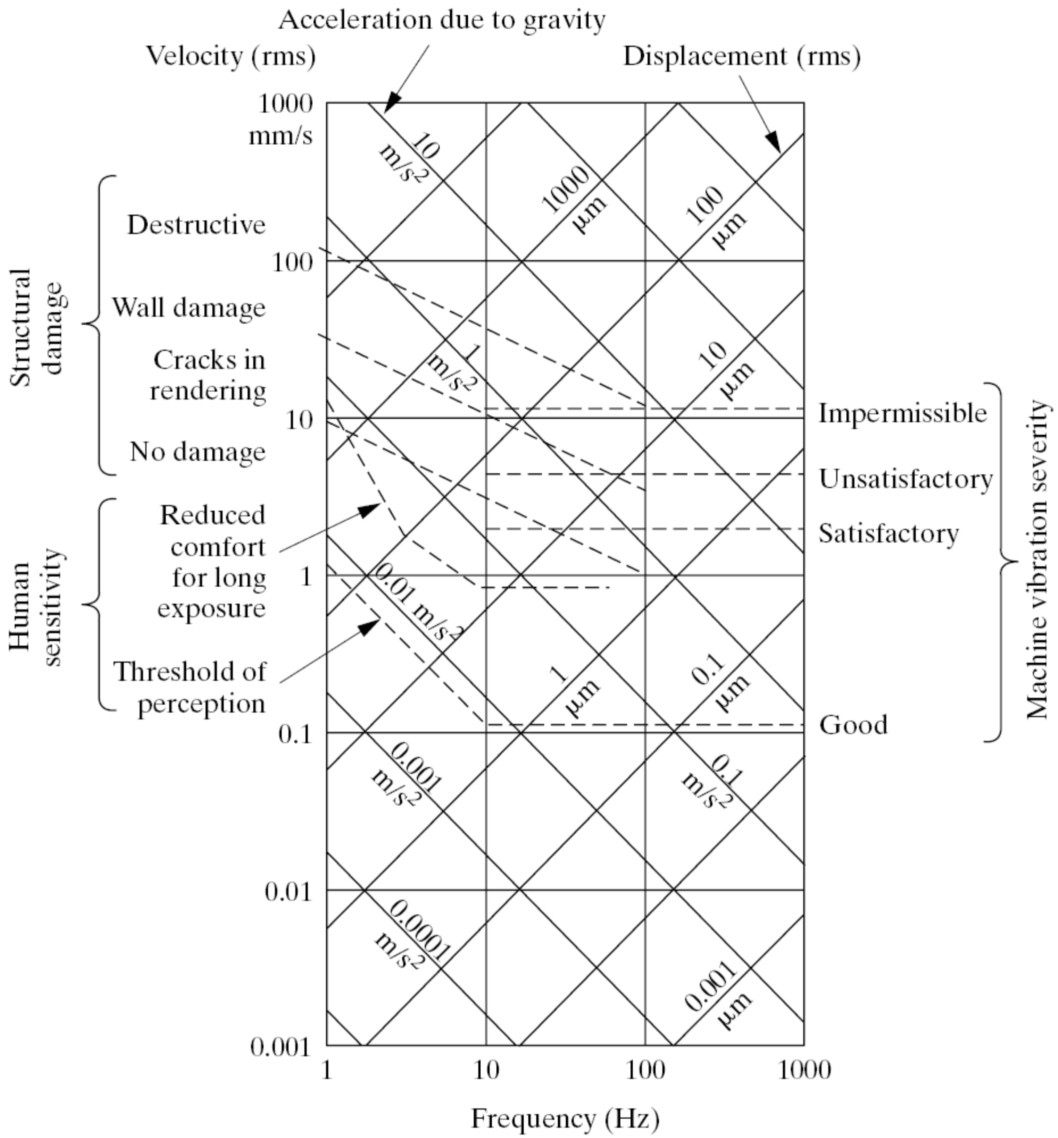
$$x_{rms} = \left[\lim_{T\to\infty} \frac{1}{T} \int_0^T x^2(t)\,dt \right]^{1/2}$$

$$x(t) = A \sin \omega_n t \quad \Rightarrow \quad \bar{x}^2 = \lim_{t\to\infty} \frac{1}{T} \int_0^T (A^2 \sin^2 \omega_n t)\,dt = \frac{A^2}{2} \quad \Rightarrow \quad \bar{x} = \frac{A}{\sqrt{2}}$$

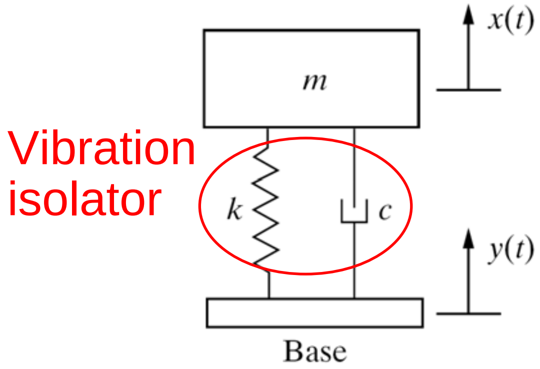
$$v(t) = \dot{x}(t) = A\omega \cos \omega_n t \quad \Rightarrow \quad \bar{v} = \frac{A\omega_n}{\sqrt{2}}$$

$$a(t) = \ddot{x}(t) = -A\omega^2 \sin \omega_n t \quad \Rightarrow \quad \bar{a} = \frac{A\omega_n^2}{\sqrt{2}}$$

Acceptable level of vibration (2)



Vibration isolation (moving base)



Objective:

To isolate a device from the source of vibration (moving base).

(To reduce vibration of machine transmitted through moving base.)

Applications:

Automobile suspension

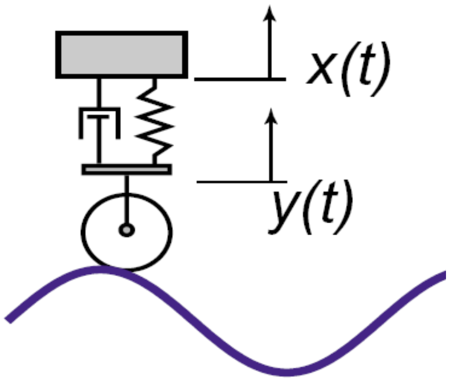
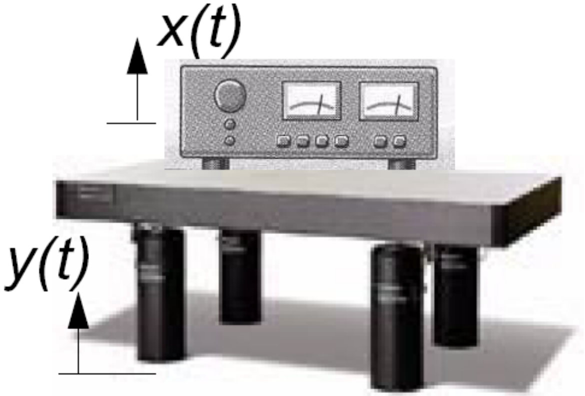
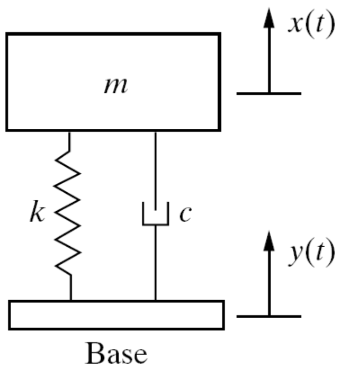


Table isolator



Base excitation (review)



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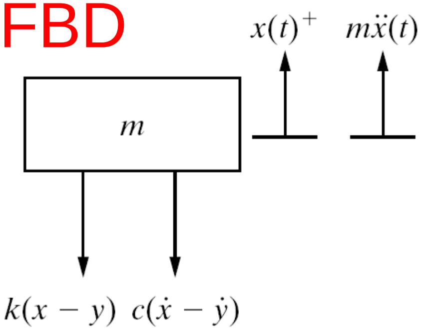


EOM $$m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$$

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = 2\zeta\omega_n\dot{y} + \omega_n^2 y$$

Harmonic motion of the base

$$y(t) = Y\cos\omega t = \mathrm{Re}[Ye^{j\omega t}]$$

The response in complex form is

$$z_p(t) = Ze^{j\omega t}$$

Substituting $z_p(t)$ into EOM yields

$$Z = \frac{\omega_n^2 + (2\zeta\omega\omega_n)j}{-\omega^2 + \omega_n^2 + (2\zeta\omega\omega_n)j}Y = \left[\frac{1 + (2\zeta r)j}{1 - r^2 + (2\zeta r)j}\right]Y \quad ; \quad r = \omega/\omega_n$$

Displacement transmissibility (1)

From $$Z = \left[\frac{1 + (2\zeta r) j}{1 - r^2 + (2\zeta r) j} \right] Y = T(\omega) Y$$

$$T(\omega) = \frac{Output}{Input} = \frac{Z}{Y} = \frac{1 + j(2\zeta r)}{1 - r^2 + j(2\zeta r)}$$

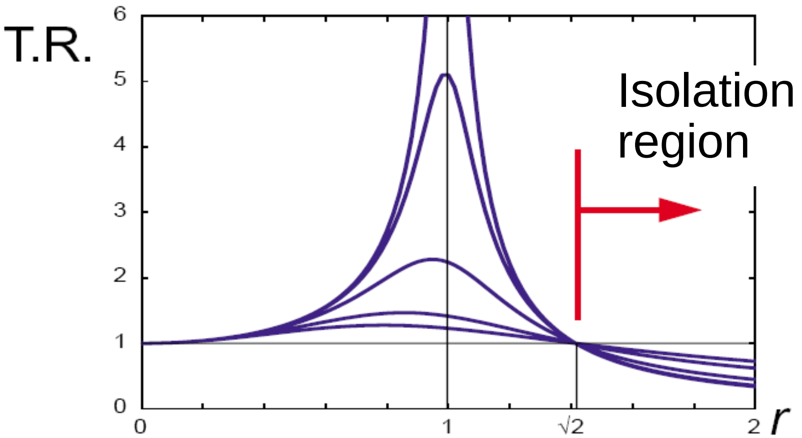
Displacement transmissibility (T.R.)

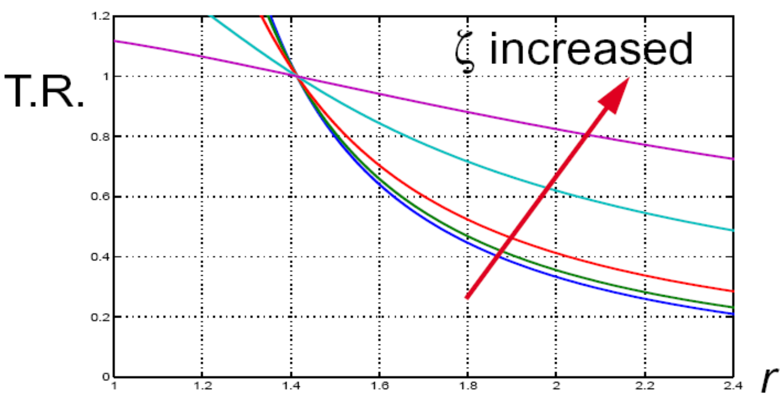
$$\left| T(\omega) \right| = \left| \frac{Z}{Y} \right| = \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

Tells how motion is transmitted from the base to the mass at various driving frequencies

Displacement transmissibility (2)

$$|T(\omega)| = \left|\frac{Z}{Y}\right| = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

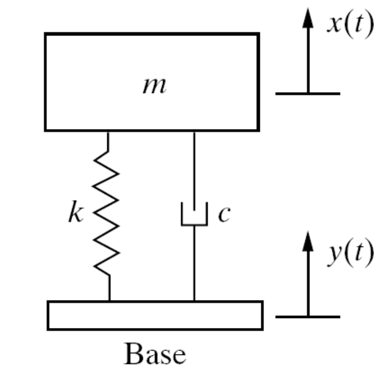




- r = 0, T.R. = 1
- Isolation occurs when $r > \sqrt{2}$
- To increase r (decrease ω_n), m or k are changed
- In the isolation region, T.R. decreases with ζ decreased

In addition to T.R., another design criteria such as transmitted forces and installation space need to be considered.

Force transmissibility (1)



EOM $\quad m\ddot{x} + c\dot{x} + kx = c\dot{y} + ky$

Force transmitted to the mass

$$F(t) = k(x-y) + c(\dot{x}-\dot{y}) = -m\ddot{x}$$

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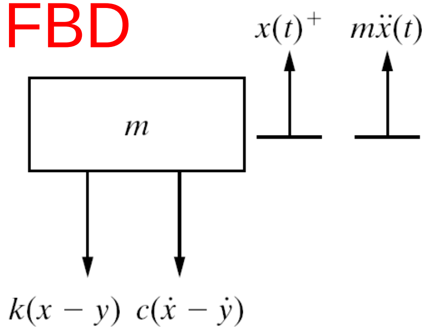


$$F(t) = m\omega^2 X \cos(\omega t + \theta)$$

$$= m\omega^2 Y |T(\omega)| \cos(\omega t + \theta)$$

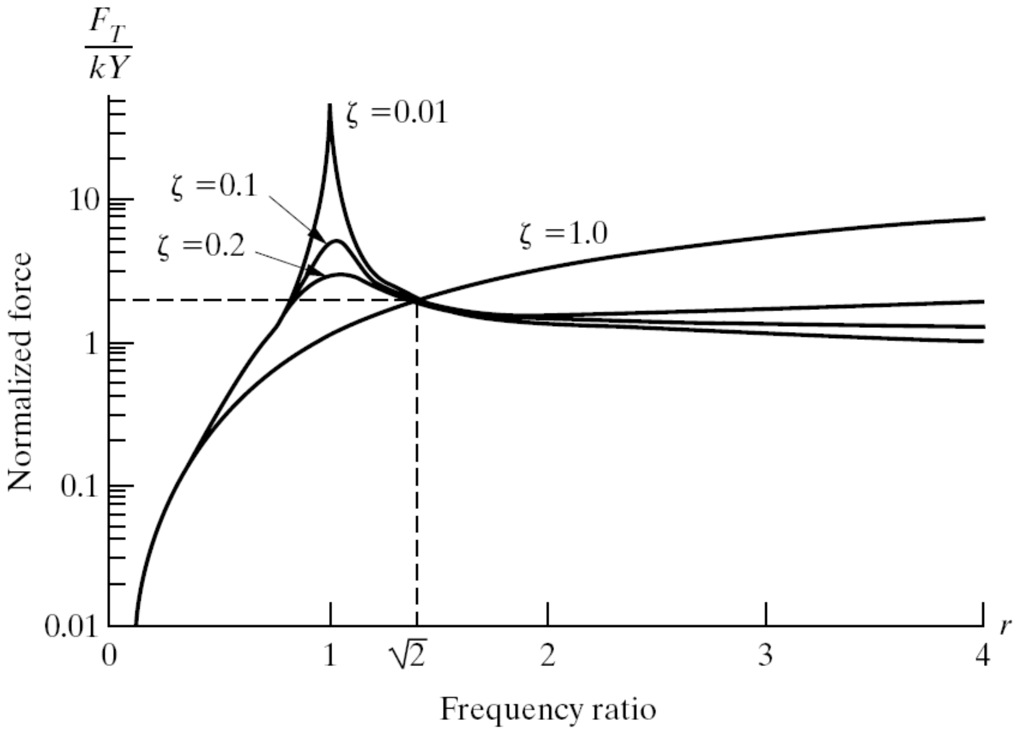
$$= F_T \cos(\omega t + \theta)$$

Force transmissibility

$$\frac{F_T}{kY} = r^2 \sqrt{\frac{1 + (2\zeta r)^2}{(1 - r^2)^2 + (2\zeta r)^2}}$$

Tells how much force is transmitted from the base to the mass at various driving frequencies.

Force transmissibility (2)

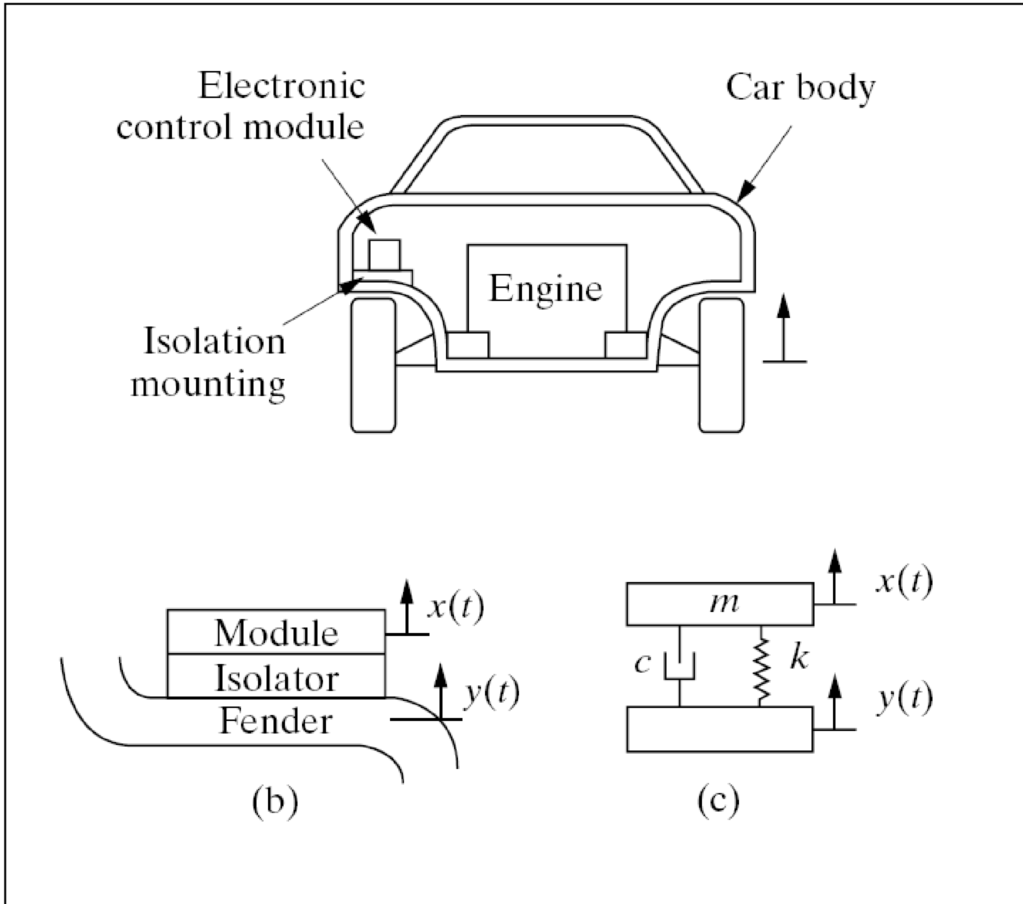


- The force transmitted does not necessarily fall off for r > $\sqrt{2}$
- The force transmitted increase dramatically for $r > \sqrt{2}$, as the damping increases.

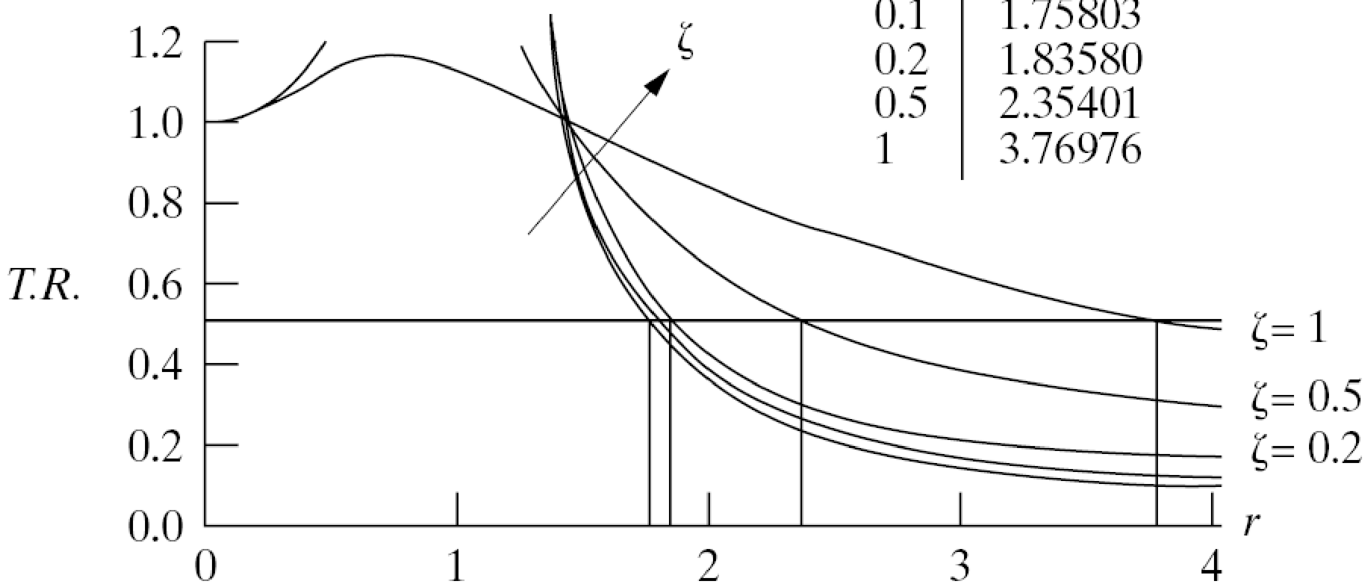
Example 5.2.1 (1)

To protect the electronic control module from fatigue and breakage, it is desirable to isolate the module from the vibration induced in the car body by road and engine vibration.

- mass of the module 3kg.
- $y(t) = 0.\ 01\sin(35t)$ m.
- Desire to keep the displacement of module less than 0.005 m at all times.
- Design values for isolator, calculate the magnitude of transmitted force.



Example 5.2.1 (2)



ζ	r
0.01	1.73231
0.05	1.73855
0.1	1.75803
0.2	1.83580
0.5	2.35401
1	3.76976

Conclusion (Design problem)

<u>Given</u>: m, ω, X, Y

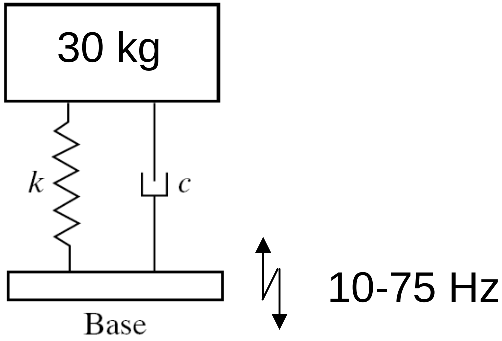
1. Calculate T.R.
2. Select ζ

   - calculate ω_n,

   - calculate k, c
3. Select k, c from catalogue
4. Calculate FT
5. Calculate static deflection (used space)

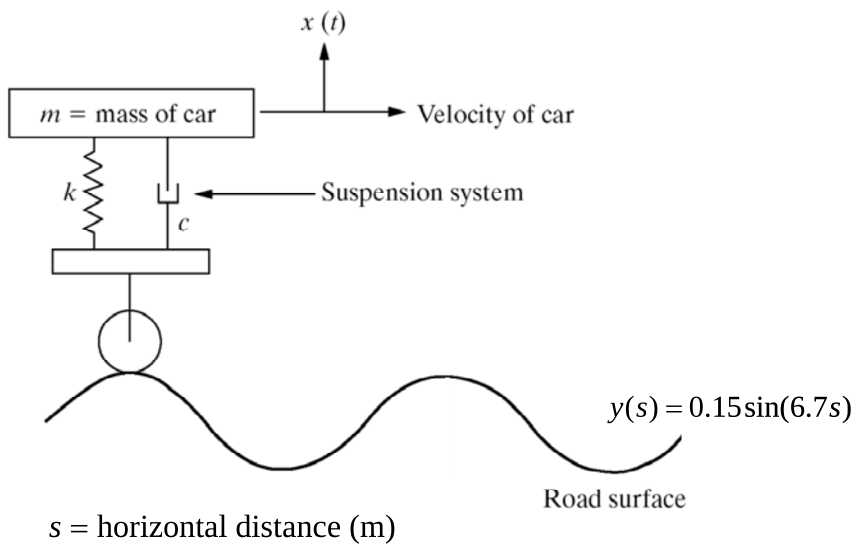
Example (1)

A sensitive electronic system, of mass 30 kg, is supported by a spring-damper system on the floor of a building that is subject to a harmonic motion in the frequency range 10-75 Hz. If the damping ratio of the suspension is 0.25, determine the stiffness of the suspension if the amplitude of vibration transmitted to the system is to be less than 15 percent of the floor vibration over the given frequency range.

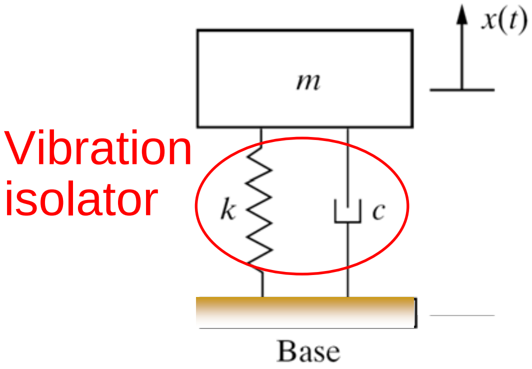


Example (2)

Design the suspension of an automobile such that the maximum vertical acceleration felt by the driver is less than 2g at all speeds between 65 and 130 km/h while traveling on a road whose surface varies sinusoidally as $y(s) = 0.15\sin(6.7s)$ meter where s is the horizontal distance in meter. The mass of the automobile, with the driver, is 680 kg and the damping ratio of the suspension is to be 0.05. Use a single degree of freedom model for the automobile.



Vibration isolation (Fixed base)



Objective:

To isolate the source of vibration (machine) from the other system.

To reduce vibration (force) transmitted from the machine to base.

Applications:

Washing machine

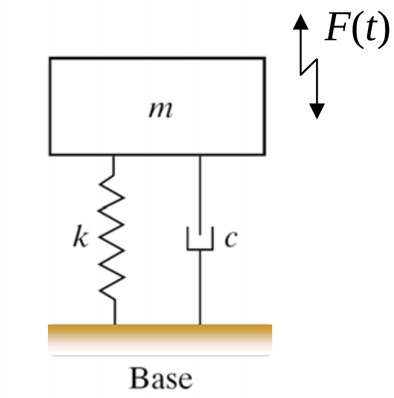
Generator or the other machines

Smaller centrifugal fans can be directly mounted on isolators. (Fan shown is a Model 18-BISW Arr. 10)

Larger centrifugal fans and all sizes of Arr. 1 or 3 fans are typically mounted on structural bases. The fan shown in this photo is mounted on a structural base with height saving brackets and free standing springs. (Fan shown is Model 30 AFDW, Arr. 3)

Coil spring isolator with integral viscous damping unit.

Force transmissibility (1)



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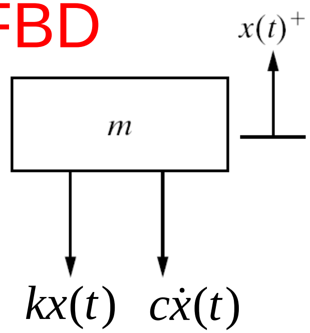


EOM $$m\ddot{x} + c\dot{x} + kx = F_0 \cos \omega t$$

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = \frac{F_0}{m} \cos \omega t$$

The steady-state response of EOM is

$$x(t) = X \cos(\omega t - \phi)$$

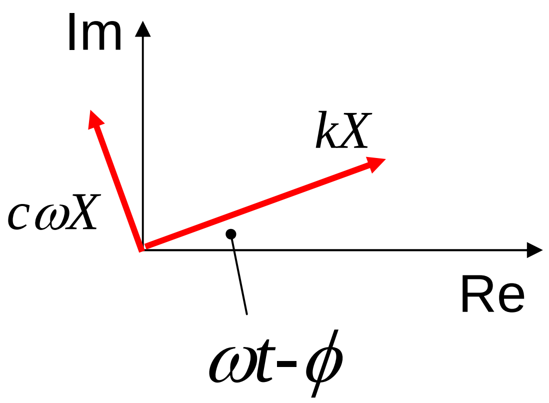
Where $$X = \frac{F_0 / k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} \quad ; \quad r = \omega / \omega_n$$

Force transmitted to base is

$$F_T(t) = kx(t) + c\dot{x}(t)$$

$$F_T(t) = kX \cos(\omega t - \phi) + c\omega X \sin(\omega t - \phi)$$

Force transmissibility (2)



$$F_T(t) = kX\cos(\omega t-\phi) + c\omega X\sin(\omega t-\phi)$$

Magnitude of $F_T(t)$ is

$$|F_T(t)| = \sqrt{(kX)^2 + (c\omega X)^2} = X\sqrt{k^2 + c^2\omega^2}$$

$$|F_T(t)| = \frac{F_0/k}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}\sqrt{k^2 + c^2\omega^2}$$

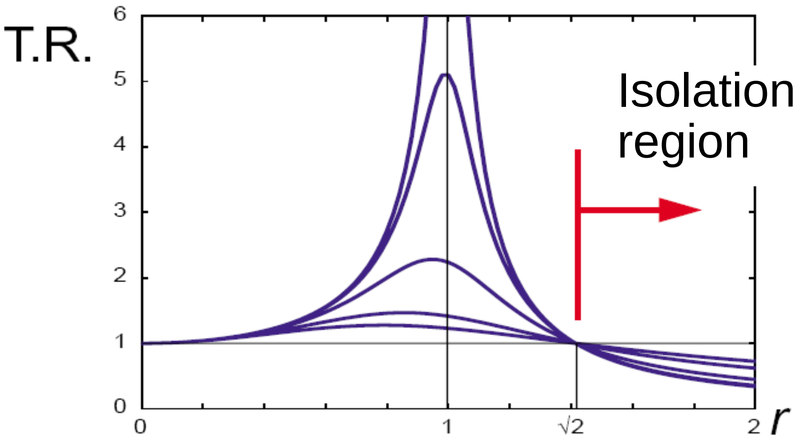
Force transmissibility (T.R.)

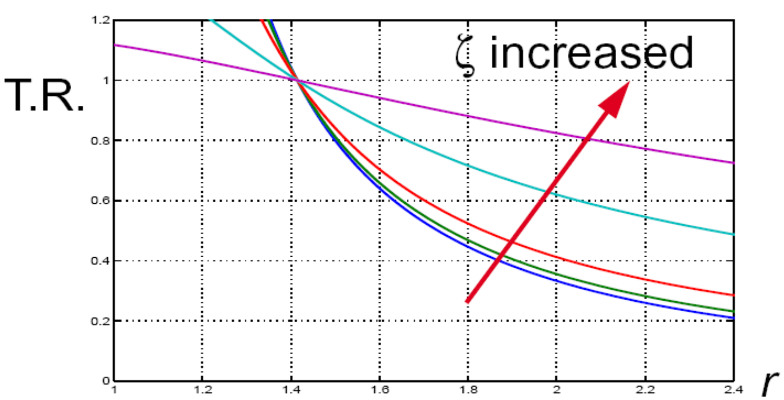
$$\left|\frac{F_T(t)}{F_0}\right| = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}}$$

Tells how much force is transmitted from the machine to the base at various driving frequencies.

Force transmissibility (3)

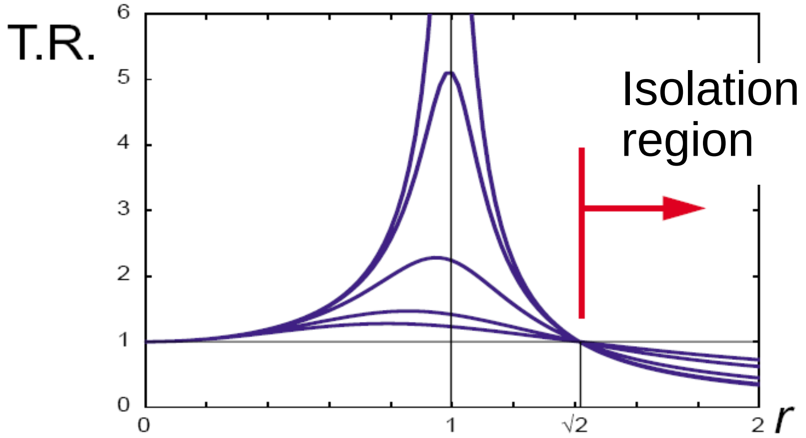
$$\left|\frac{F_T(t)}{F_0}\right| = \sqrt{\frac{1+(2\zeta r)^2}{(1-r^2)^2+(2\zeta r)^2}}$$

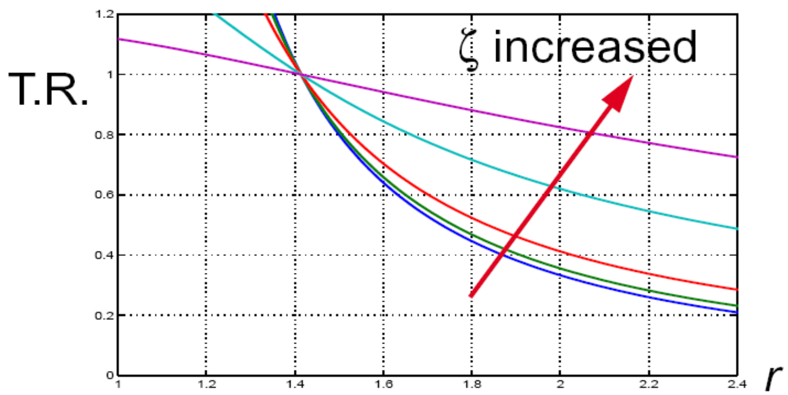




- Disp. T.R. (moving base) and Force T.R. (fixed base) have the same form
- Design consideration for both cases are the same
- Isolation occurs when *r* $> \sqrt{2}$
- To increase r (decrease ω_n), m and *k* are changed
- In the isolation region, T.R. decreases with ζ decreased
- Some damping is also desirable to reduce vibration at resonance

Force transmissibility (4)





For a large freq. ratio (r > 3) and small damping (ζ < 0.2)

T.R. is not effected by damping

Damping term can be neglected in common design

$$T.R. \approx \frac{1}{r^2 - 1}$$

Construction of design curves (1)

$$T.R. \approx \frac{1}{r^2 - 1} \qquad (r > 3,\ \zeta < 0.2)$$

Reduction in T.R.: R = 1 – T.R.

$$r = \frac{\omega}{\sqrt{k/m}} = \sqrt{\frac{2-R}{1-R}}$$

$$n = \frac{60}{2\pi}\sqrt{\frac{g(2-R)}{\Delta(1-R)}} = 29.9093\sqrt{\frac{2-R}{\Delta(1-R)}}$$

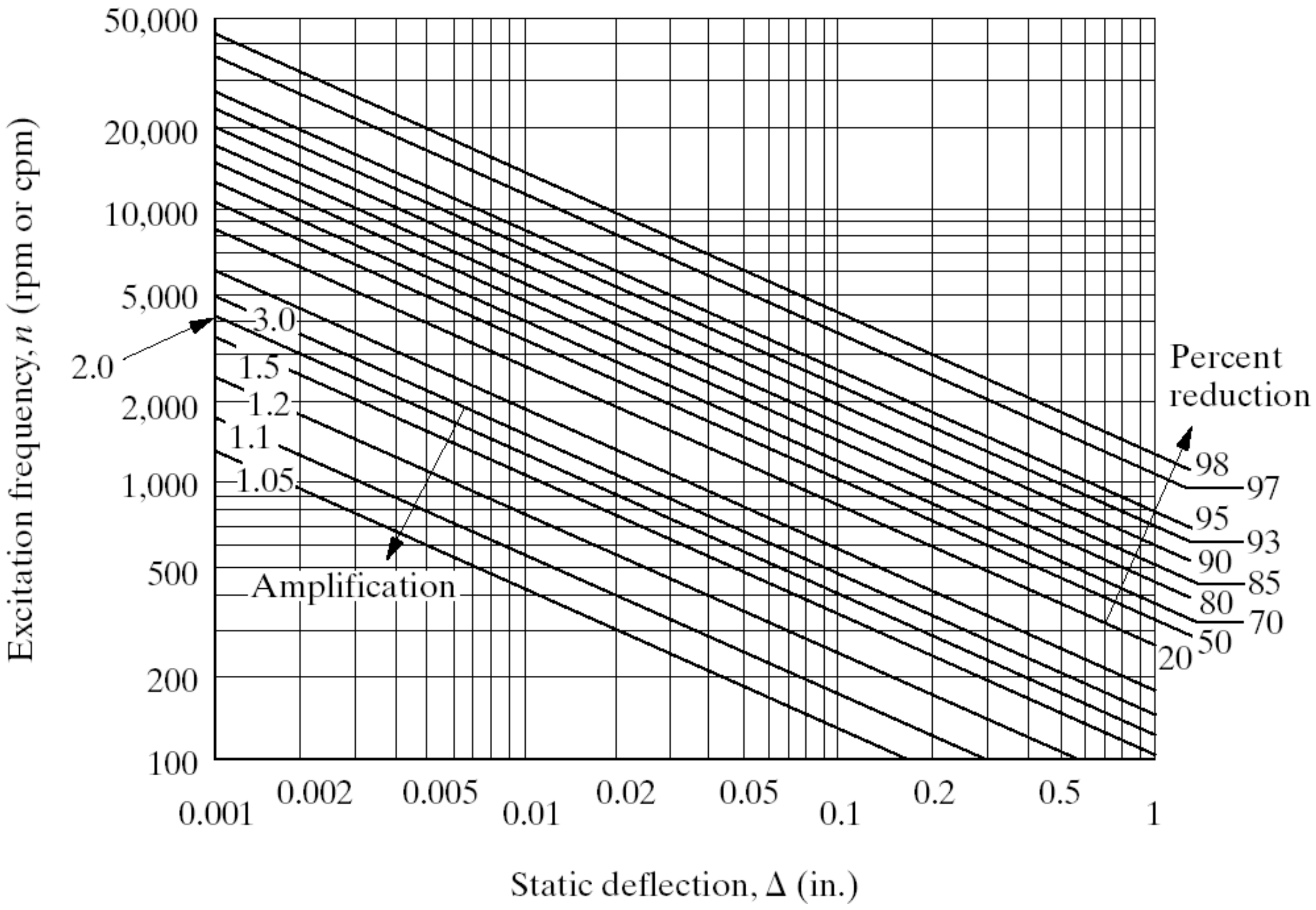
n = speed in rpm

$k = mg/\Delta$

Δ is static deflection

$$\log n = -\frac{1}{2}\log\Delta + \log\left(29.9093\sqrt{\frac{2-R}{1-R}}\right)$$

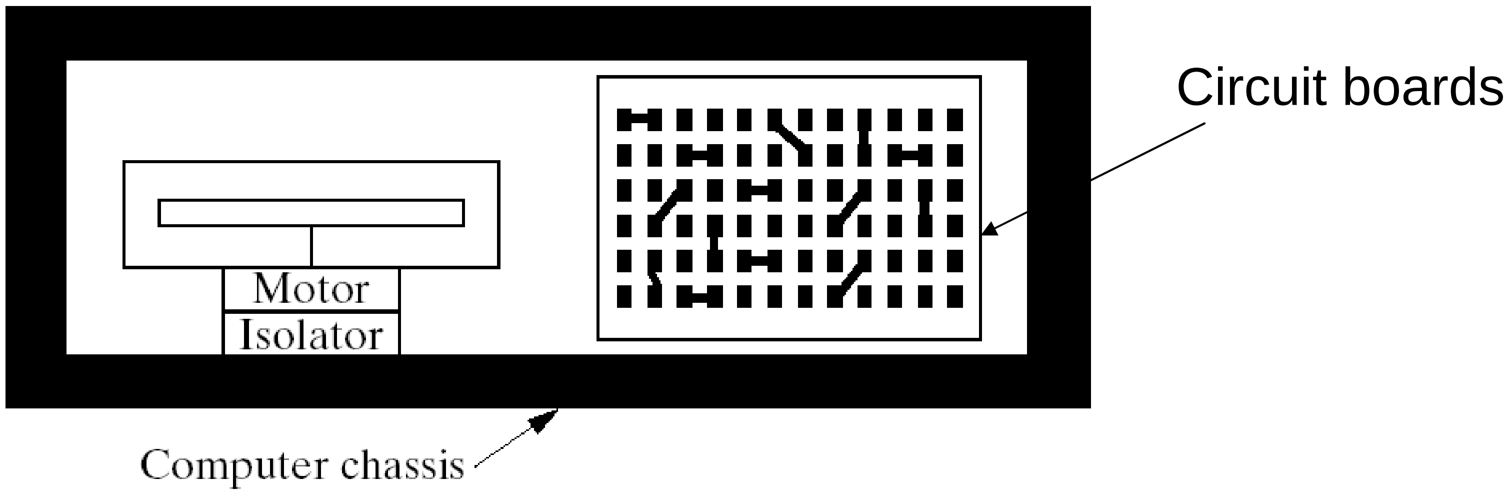
Construction of design curves (2)



$$\log n = -\frac{1}{2}\log\Delta + \log\left(29.9093\sqrt{\frac{2-R}{1-R}}\right)$$

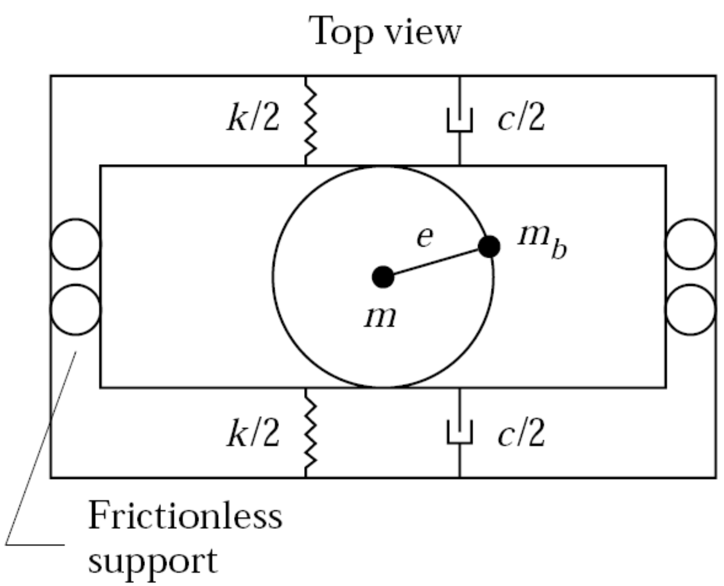
Example 5.2.2

The disk drive motor is mounted to the computer chassis through an isolation pad (spring). The motor has a mass of 3 kg and operates at 5000 rpm. Calculate the value of stiffness of the isolator needed to provide a 95% reduction in force transmitted to the chassis (considered as ground). How much clearance is needed between the motor and the chassis.



Example: washing machine

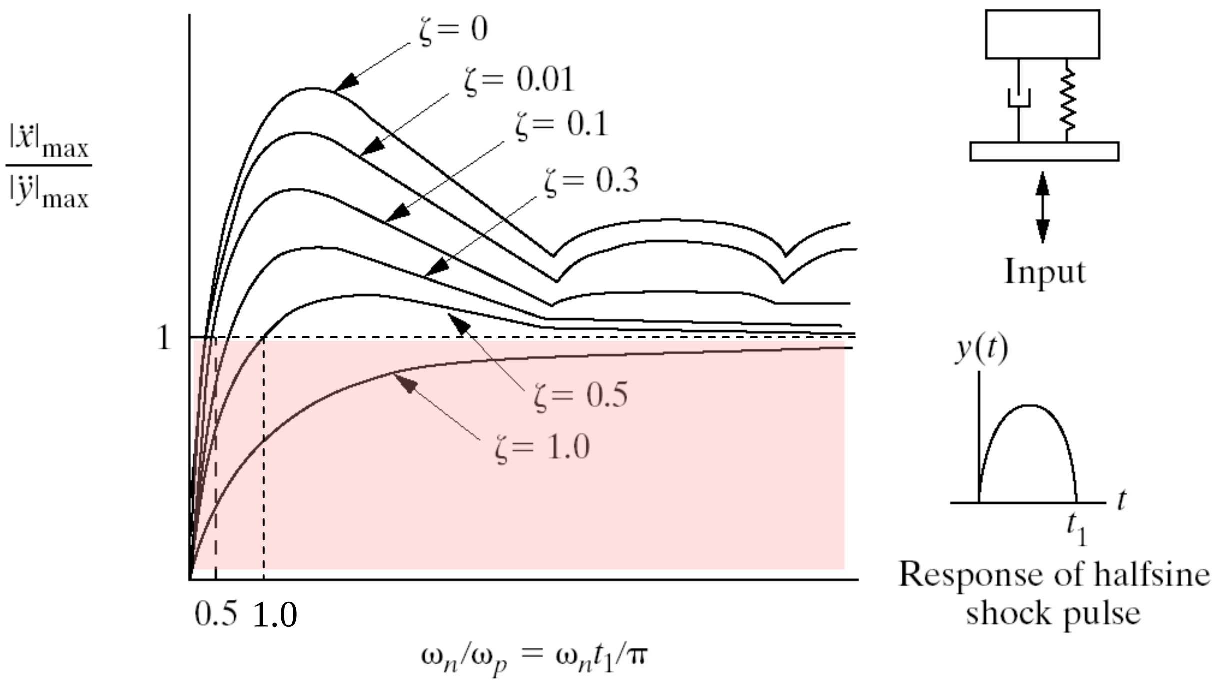
A model of a washing machine is illustrated in the figure. A bundle of wet clothes form a mass of 10 kg (m_0) and causes a rotating unbalance. The rotating mass id 20 kg (including m_0) and the diameter of the washer basket ($2e$) is 50 cm. Assume that the spin cycle rotates at 300 rpm. Let k be 1000 N/m and ζ =0.01. (a) Calculate the force transmitted to the sides of the washing machine. (b) The quantities m, m_0 e and ω are all fixed by the previous design. Design the isolation system so that the force transmitted to the side of the washing machine is less than 100 N.



Example: Design for vibration isolation

It has been found that a printing press, of mass 300 kg and operating speed 3000 rpm, produces a repeating force of 30,000 N when attached to a rigid foundation. Find a suitable viscously damped isolator to satisfy the following requirements: (a) the static deflection should be as small as possible; (b) the steady-state amplitude should be less than 2.5 mm; (c) the amplitude during start-up conditions should not exceed 20 mm; and (d) the force transmitted to the foundation should be less than 10,000 N.

Shock Isolator



- Reduction of the acc. through isolation occurs only if the T.R. falls below 1.
- Shock isolation enforces a boundary on the stiffness.
- Increasing the damping greatly reduces the max. acc.