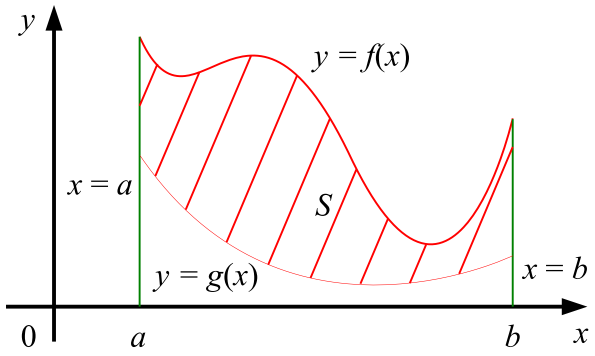
2301107 Calculus I

7. Applications of the integrations

Outline

7.1. Areas between positive curves

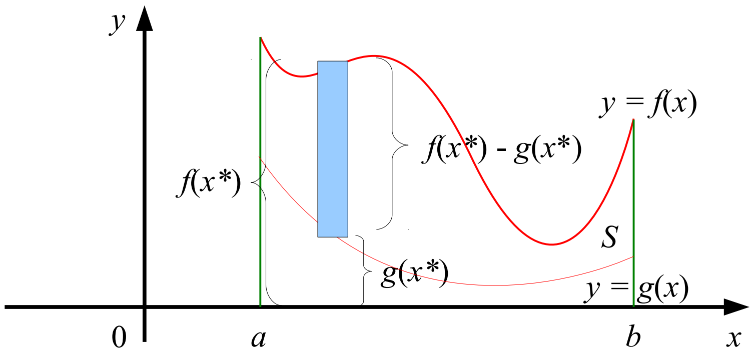
- Find the area of the region S that lies between positive functions f and g from a to b.



Areas between curves

- Using Riemann sum on the range that $f(x) \geq g(x)$.

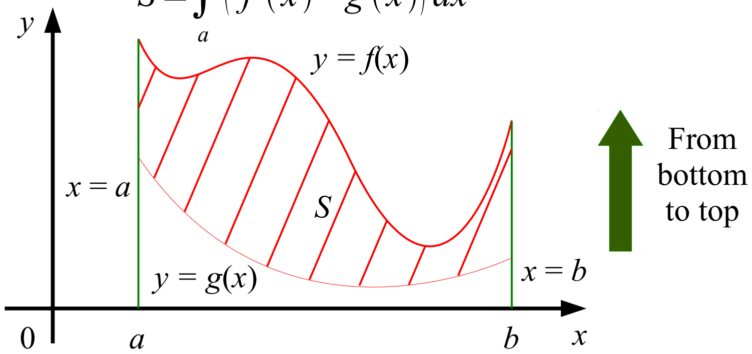
$$S = \lim_{n\to\infty} \sum_{i=1}^{n} \left(f(x_i^*) - g(x_i^*) \right) \Delta x$$



Areas between curves by x-axis

- The area S bounded by $y = f(x)$, $y = g(x)$, $x=a$, $x=b$ where f and g are continuous and $f(x) \geq g(x)$.

$$S = \int_a^b \left(f(x) - g(x) \right) dx$$



Student note

1. Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$ and bounded on the sides by $x = 0$ and $x = 1$.

Student note

2. Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

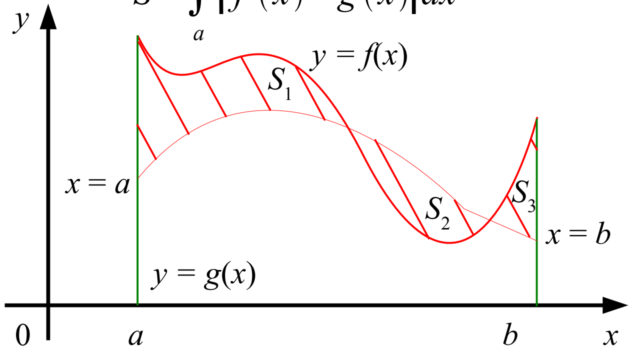
Student note

3. Find the area of the whale-shaped region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

Areas between general curves

- The area between the curves $y = f(x)$ and $y = g(x)$ from $x = a$ to $x = b$ is

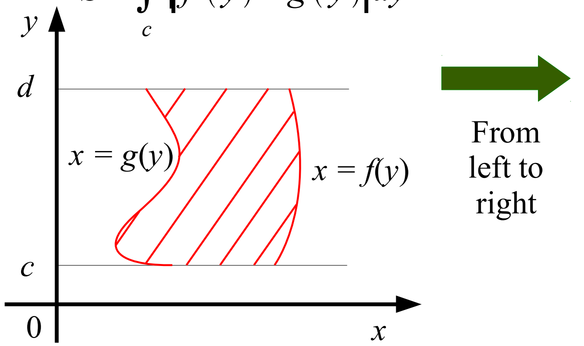
$$S=\int_a^b \left|f(x)-g(x)\right|dx$$



Areas between curves by y-axis

- Some regions are best treated by regarding x as a function of y. The area between $x=f(y)$ and $x=g(y)$ is

$$S=\int_c^d \left|f(y)-g(y)\right|dy$$



Student note

4. Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

Student note

5. Sketch the region bounded by $y = \dfrac{1}{x}$, $y = \dfrac{1}{x^2}$, $x = 2$ and find the area of the region.

Student note

6. Find the area enclosed by the given curves
   6.1. $y = x$, $y = 9 - x^2$, $x = -1$, $x = 2$
   6.2. $y = \ x$, $y = x^2$

Student note

7. Find the area enclosed by the given curves
   7.1. $y = x$, $y = |x^3|$
   7.2. $y = |x|$, $y = x^2 - 2$

Student note

8. Evaluate the integral and interpret it as the area of a region.

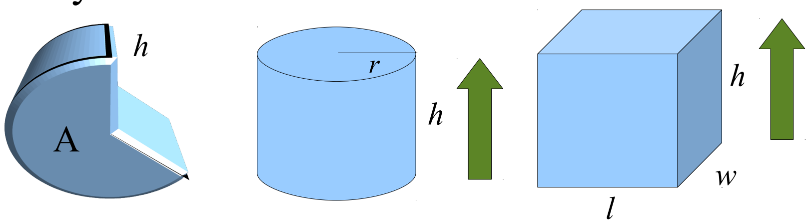
$$\int_{-1}^{1} |x^3 - x|\, dx$$

Volumes

- Cylinder is the type of solid that can be generated from the base and the direction of its height. Hence, the volume of the cylinder is

$$V = Ah$$

where A is the area of the base and h is the height of the cylinder.



$V = Ah$ $\quad$ $V = \pi r^2 h$ $\quad$ $V = lwh$

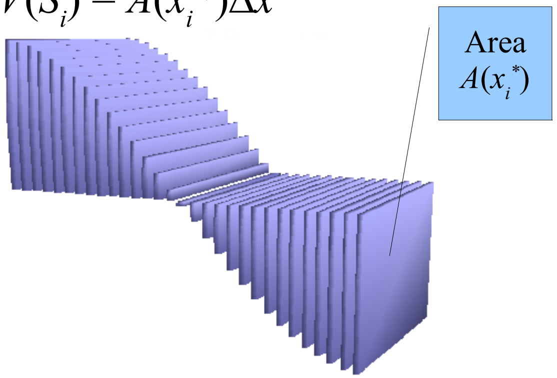
Solid

- A volume of the non-cylindrical solid S can be approximated using the cylinder of the cross-section of S. Each slice has the volume

$$V(S_i) = A(x_i^*)\Delta x$$

Therefore,

$$V = \sum_{i=1}^{n} A(x_i^*)\Delta x$$



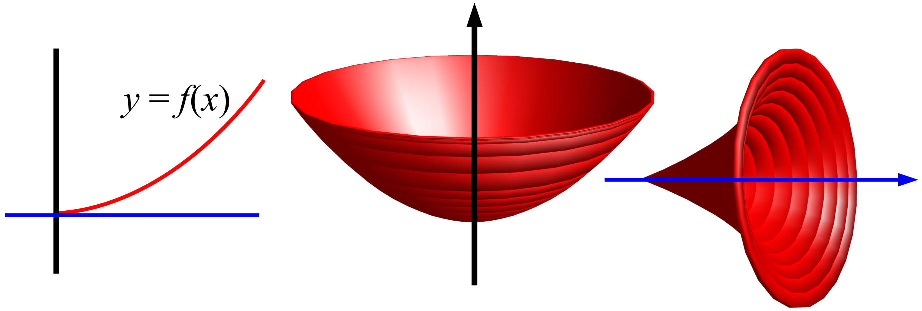
Definition of Volume

- Definition: Let S be a solid that lies between a and b. If the cross-sectional area of S in the plane P_x, through x and perpendicular to the x-axis, is $A(x)$, where A is a continuous function, then the **volume** of S is

$$V = \lim_{n\to\infty} \sum_{i=1}^{n} A(x_i^*)\Delta x = \int_a^b A(x)\,dx$$

7.2. Solid of revolution

- The solid can be formed by rotating the curve around a given axis called solid of revolution.



Rotate about y-axis Rotate about x-axis

Student note

9. Show that the volume of a sphere of radius r is

$$V = \frac{4}{3}\pi r^3$$

Student note

10. Find the volume of the solid obtained by rotating about the x-axis the region under $y = \sqrt{x}$ on [0, 1].

Student note

11. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$ and $x = 0$ about the y-axis.

Student note

12. Find the volume of the solid that formed by rotating the enclosed region about x-axis of the curves $y = x$ and $y = x^2$.

Student note

13. Find the volume of the solid that formed by rotating the enclosed region about $y = 2$ of the curves $y = x$ and $y = x^2$.

Student note

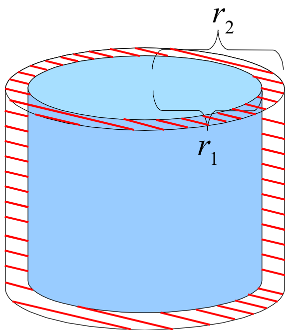
14. Find the volume of the solid that formed by rotating the enclosed region about x = -1 of the curves $y = x$ and $y = x^2$.

Student note

15. Find the volume of a pyramid whose base is a square with side L and whose height is h.

7.3. Volume by Cylindrical Shells

- The method of cylindrical shells computes the volume by considered the difference of inner cylinder and outer cylinder.



$$V = V_1 - V_2 = \pi r_2^2 h - \pi r_1^2 h$$
$$= \pi(r_2 + r_1)(r_2 - r_1)h$$
$$= 2\pi(r_2 + r_1)/2\ h(r_2 - r_1)$$

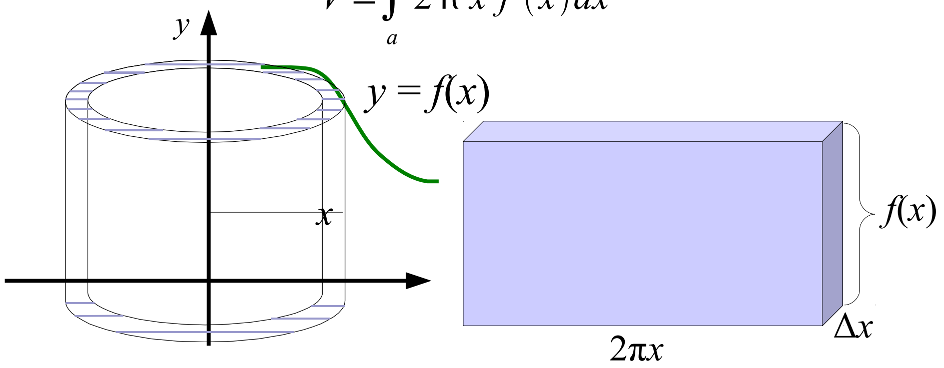
$$V = 2\pi r h \Delta x$$

$$r = \frac{r_2 + r_1}{2}$$

Volume of the solid

- The volume of the solid obtained by rotating about the *y*-axis the region under $y = f(x)$ from a to b is

$$V = \int_a^b 2\pi x f(x) dx$$



Student note

16. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

Student note

17. Find the volume of the solid obtained by rotating about the y-axis the region between $y = x$ and $y = x^2$.

Student note

18. Use cylindrical shells to find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1.