

2301107 Calculus I

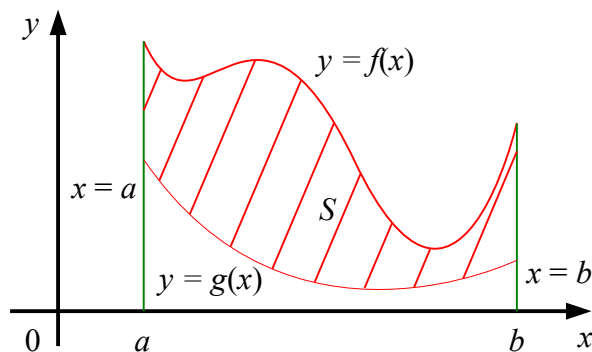
7. Applications of the integrations

Outline

- 7.1. Area between curves
- 7.2. Volume of the solid by rotation
- 7.3. Volume by cylindrical shells

7.1. Areas between positive curves

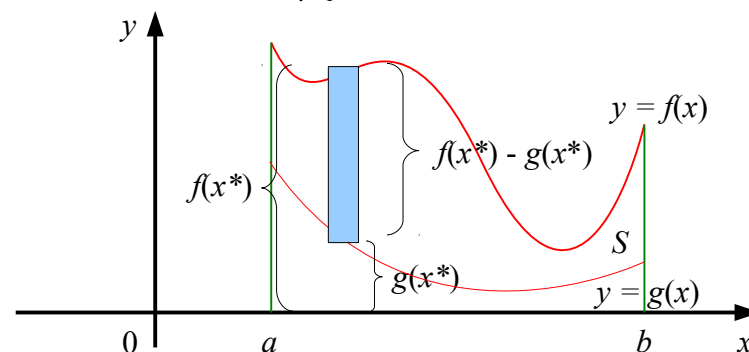
- Find the area of the region S that lies between positive functions f and g from a to b .



Areas between curves

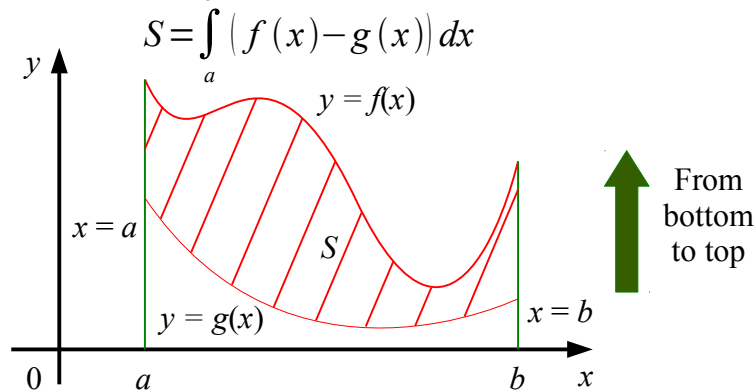
- Using Riemann sum on the range that $f(x) \geq g(x)$.

$$S = \lim_{n \rightarrow \infty} \sum_{i=1}^n (f(x_i^*) - g(x_i^*)) \Delta x$$



Areas between curves by x-axis

- The area S bounded by $y = f(x)$, $y = g(x)$, $x = a$, $x = b$ where f and g are continuous and $f(x) \geq g(x)$.



Student note

- Find the area of the region bounded above by $y = x^2 + 1$, bounded below by $y = x$ and bounded on the sides by $x = 0$ and $x = 1$.

Student note

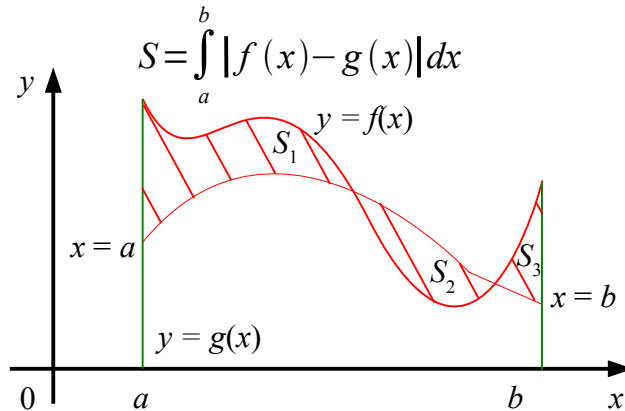
- Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

Student note

- Find the area of the whale-shaped region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

Areas between general curves

- The area between the curves $y = f(x)$ and $y = g(x)$ from $x = a$ to $x = b$ is

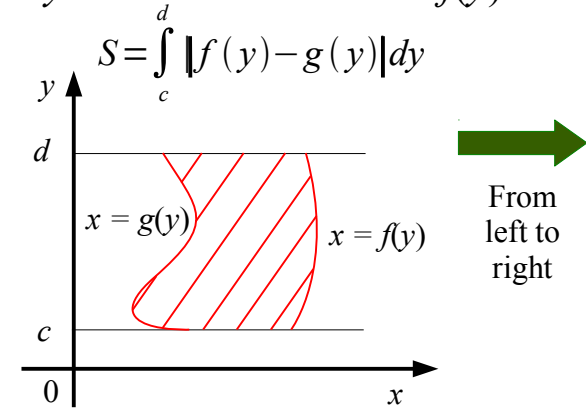


Chapter 7: Applications of the integrations

C07-9

Areas between curves by y-axis

- Some regions are best treated by regarding x as a function of y . The area between $x = f(y)$ and $x = g(y)$ is



Chapter 7: Applications of the integrations

C07-10

Student note

- Find the area enclosed by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

Chapter 7: Applications of the integrations

C07-11

Student note

- Sketch the region bounded by $y = \frac{1}{x}$, $y = \frac{1}{x^2}$, $x = 2$ and find the area of the region.

Chapter 7: Applications of the integrations

C07-12

Student note

6. Find the area enclosed by the given curves

6.1. $y = x$, $y = 9 - x^2$, $x = -1$, $x = 2$

6.2. $y = x$, $y = x^2$

Student note

8. Evaluate the integral and interpret it as the area of a region.

$$\int_{-1}^1 |x^3 - x| dx$$

Student note

7. Find the area enclosed by the given curves

7.1. $y = x$, $y = |x^3|$

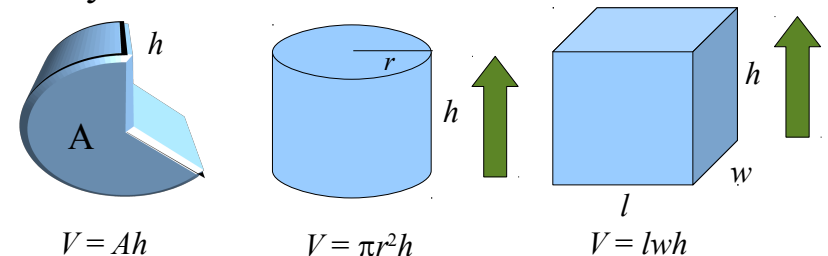
7.2. $y = |x|$, $y = x^2 - 2$

Volumes

- Cylinder is the type of solid that can be generated from the base and the direction of its height. Hence, the volume of the cylinder is

$$V = Ah$$

where A is the area of the base and h is the height of the cylinder.



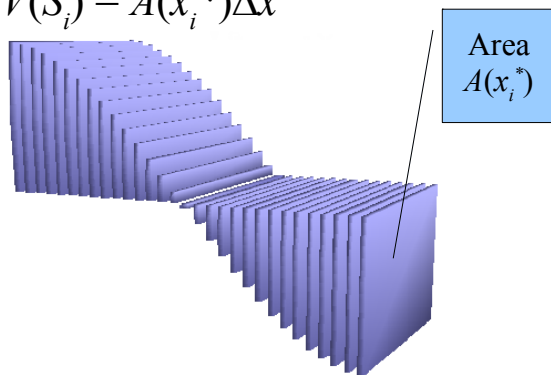
Solid

- A volume of the non-cylindrical solid S can be approximated using the cylinder of the cross-section of S . Each slice has the volume

$$V(S_i) = A(x_i^*)\Delta x$$

Therefore,

$$V = \sum_{i=1}^n A(x_i^*)\Delta x$$



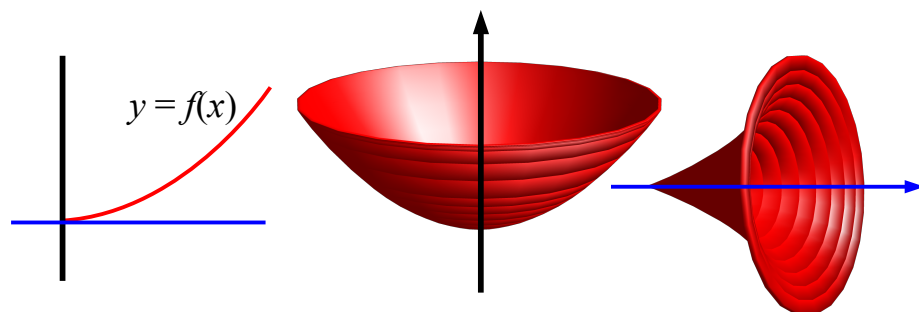
Definition of Volume

- Definition:** Let S be a solid that lies between a and b . If the cross-sectional area of S in the plane P_x , through x and perpendicular to the x -axis, is $A(x)$, where A is a continuous function, then the **volume** of S is

$$V = \lim_{n \rightarrow \infty} \sum_{i=1}^n A(x_i^*)\Delta x = \int_a^b A(x) dx$$

7.2. Solid of revolution

- The solid can be formed by rotating the curve around a given axis called solid of revolution.



Rotate about y-axis Rotate about x-axis

Student note

9. Show that the volume of a sphere of radius r is

$$V = \frac{4}{3} \pi r^3$$

Student note

10. Find the volume of the solid obtained by rotating about the x -axis the region under $y = \sqrt{x}$ on $[0, 1]$.

Student note

12. Find the volume of the solid that formed by rotating the enclosed region about x -axis of the curves $y = x$ and $y = x^2$.

Student note

11. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 8$ and $x = 0$ about the y -axis.

Student note

13. Find the volume of the solid that formed by rotating the enclosed region about $y = 2$ of the curves $y = x$ and $y = x^2$.

Student note

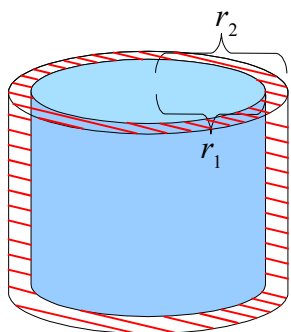
14. Find the volume of the solid that formed by rotating the enclosed region about $x = -1$ of the curves $y = x$ and $y = x^2$.

Student note

15. Find the volume of a pyramid whose base is a square with side L and whose height is h .

7.3. Volume by Cylindrical Shells

- The method of cylindrical shells computes the volume by considering the difference of inner cylinder and outer cylinder.



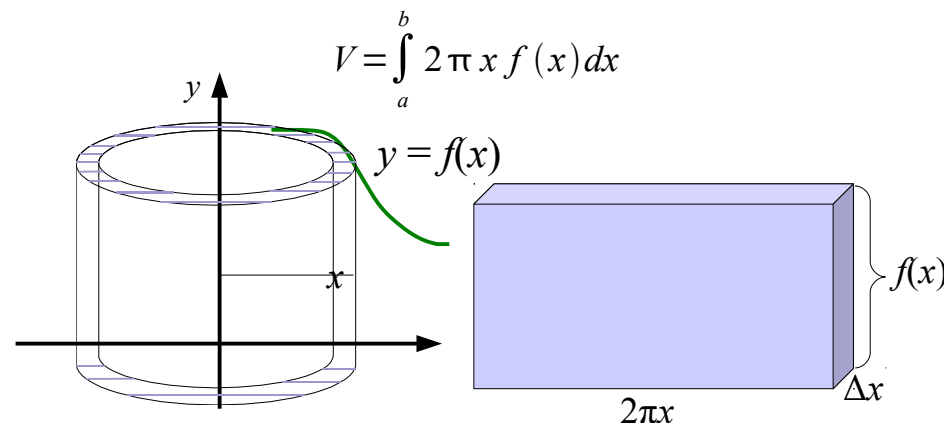
$$\begin{aligned} V &= V_1 - V_2 = \pi r_2^2 h - \pi r_1^2 h \\ &= \pi(r_2 + r_1)(r_2 - r_1)h \\ &= 2\pi(r_2 + r_1)/2 h(r_2 - r_1) \end{aligned}$$

$$V = 2\pi r h \Delta x$$

$$r = \frac{r_2 + r_1}{2}$$

Volume of the solid

- The volume of the solid obtained by rotating about the y -axis the region under $y = f(x)$ from a to b is



Student note

16. Find the volume of the solid obtained by rotating about the y -axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

Student note

17. Find the volume of the solid obtained by rotating about the y -axis the region between $y = x$ and $y = x^2$.

Student note

18. Use cylindrical shells to find the volume of the solid obtained by rotating about the x -axis the region under the curve $y = \sqrt{x}$ from 0 to 1.