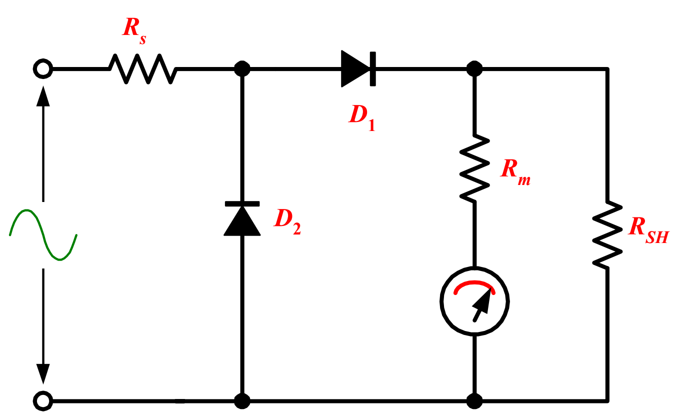
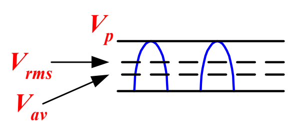
AC Voltmeter: PMMC Based

Example: A PMMC instrument has FSD of 100 μA and a coil resistance of 1700 Ω is used in the half-wave rectifier voltmeter. The silicon diode (D1) must have a minimum (peak) forward current of 100 μA. When the measured voltage is 20% of FSD. The voltmeter is to indicate 50 V_{rms} at full scale Calculate the values of R_S and R_{SH}.

Known: FSD of I_m R_m

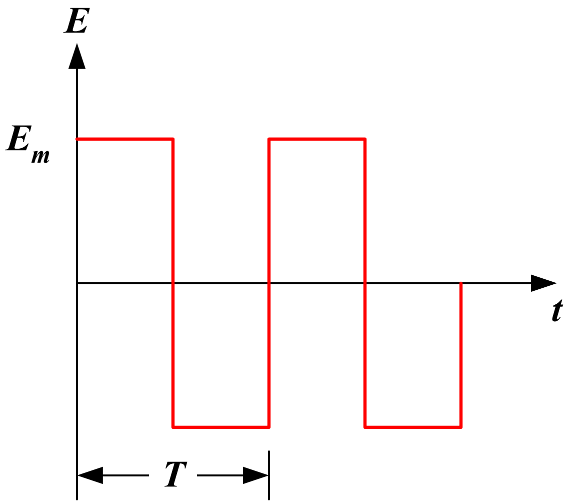
Solution: R_S = 139.5 kΩ R_{SH} = 778 Ω





AC Voltmeter: PMMC Based

Example The symmetrical square-wave voltage is applied to an average-responding ac voltmeter with a scale calibrated in terms of the rms value of a sine wave. If the voltmeter is the full-wave rectified configuration. Calculate the error in the meter indication. Neglect all voltage drop in all diodes.



Solution 11%

Bridge Circuit

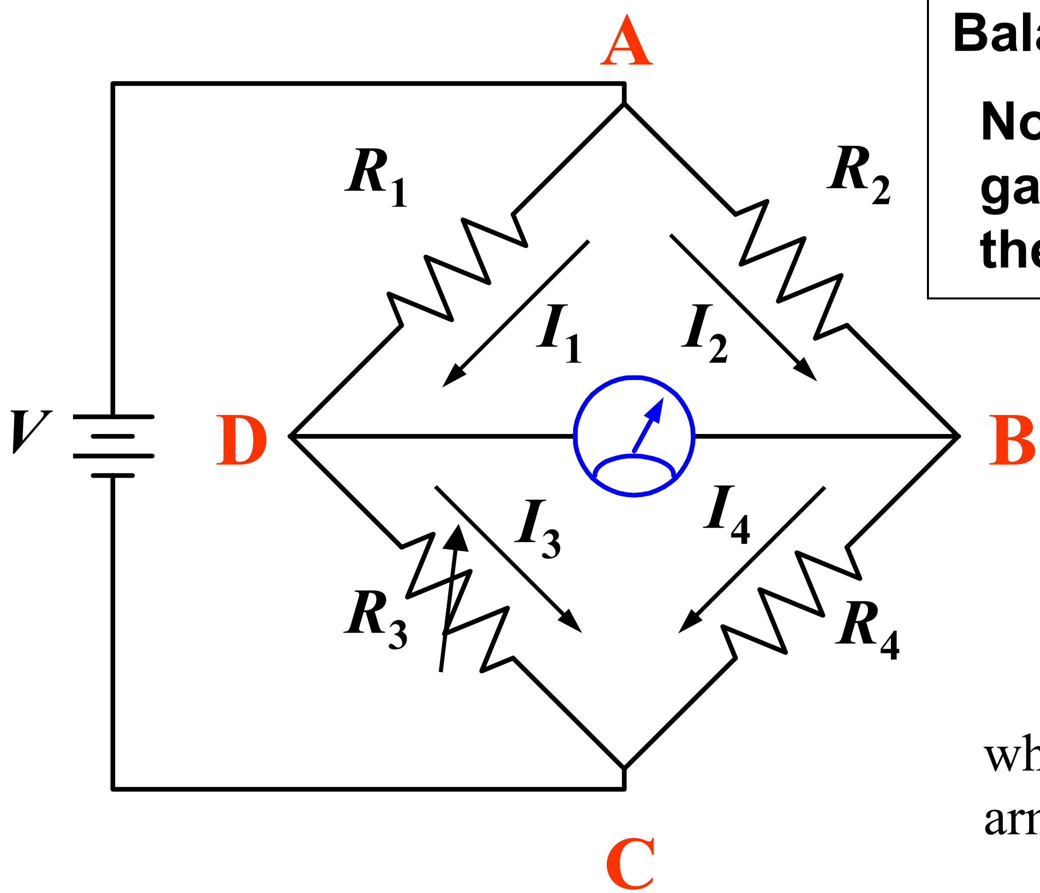
Bridge Circuit is a null method, operates on the principle of comparison. That is a known (standard) value is adjusted until it is equal to the unknown value.

Bridge Circuit

- **DC Bridge (Resistance)**
  - Wheatstone Bridge
  - Kelvin Bridge
  - Megaohm Bridge
- **AC Bridge**
  - **Inductance**
    - Maxwell Bridge
    - Hay Bridge
    - Owen Bridge
    - Etc.
  - **Capacitance**
    - Schering Bridge
  - **Frequency**
    - Wien Bridge

Wheatstone Bridge and Balance Condition

The standard resistor R_3 can be adjusted to null or balance the circuit.



Balance condition:

No potential difference across the galvanometer (there is no current through the galvanometer)

Under this condition: $V_{AD} = V_{AB}$

$$I_1 R_1 = I_2 R_2$$

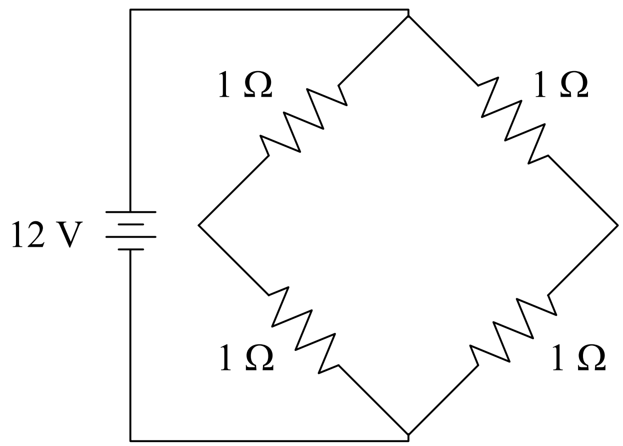
And also $V_{DC} = V_{BC}$

$$I_3 R_3 = I_4 R_4$$

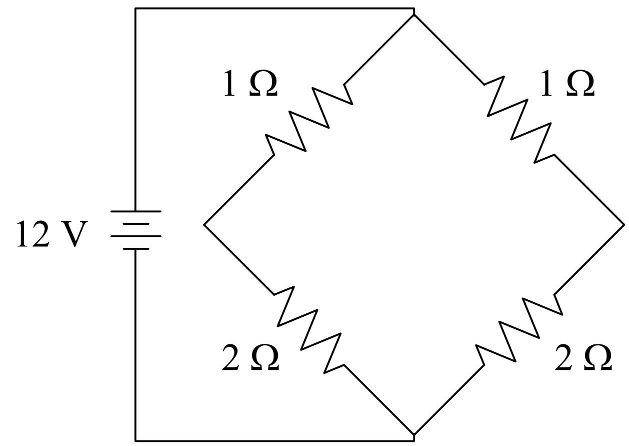
where I_1, I_2, I_3, and I_4 are current in resistance arms respectively, since $I_1 = I_3$ and $I_2 = I_4$

$$\frac{R_1}{R_3} = \frac{R_2}{R_4} \quad \text{or} \quad R_x = R_4 = R_3 \frac{R_2}{R_1}$$

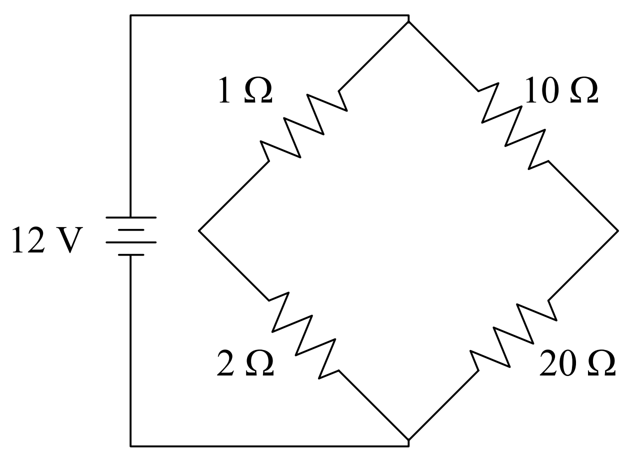
Example



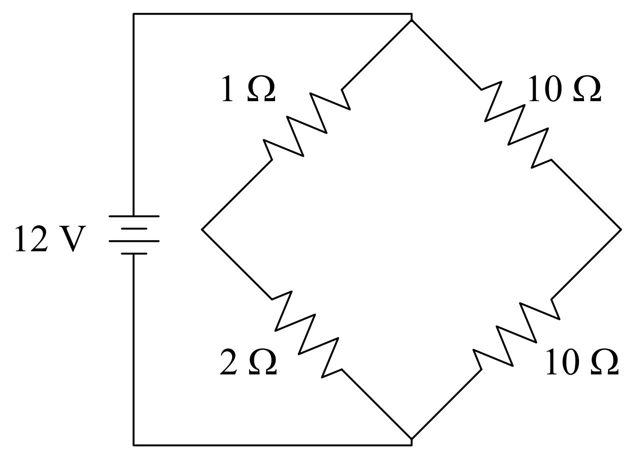
(a) Equal resistance



(b) Proportional resistance



(c) Proportional resistance



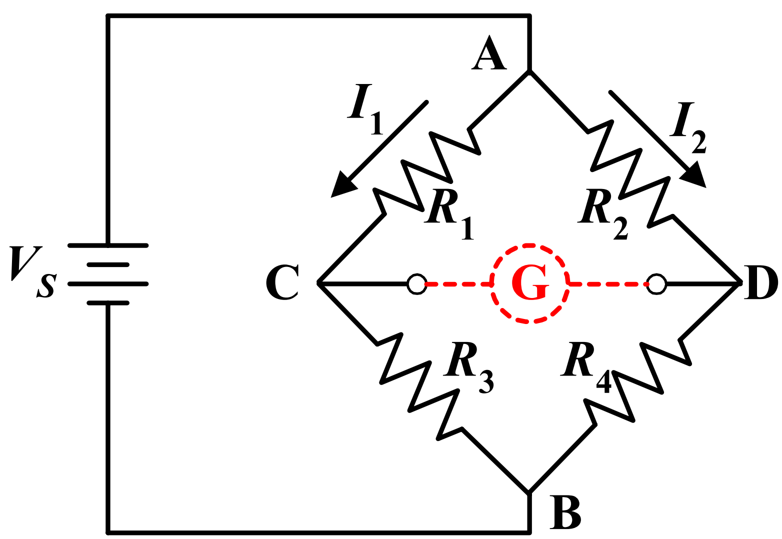
(d) 2-Volt unbalance

Sensitivity of Galvanometer

A galvanometer is use to detect an unbalance condition in Wheatstone bridge. Its sensitivity is governed by: *Current sensitivity (currents per unit defection) and internal resistance.*

consider a bridge circuit under a small unbalance condition, and apply circuit analysis to solve the current through galvanometer

Thévenin Equivalent Circuit



Thévenin Voltage (V_{TH})

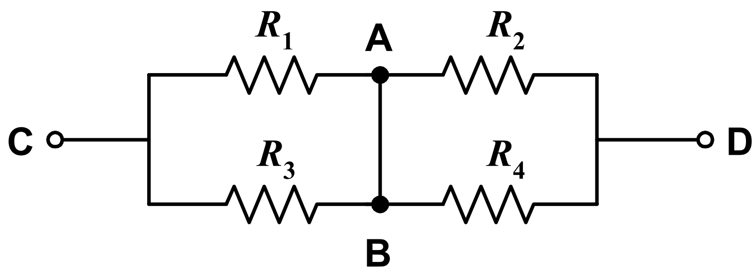
$$V_{CD} = V_{AC} - V_{AD} = I_1 R_1 - I_2 R_2$$

where $I_1 = \dfrac{V}{R_1 + R_3}$ and $I_2 = \dfrac{V}{R_2 + R_4}$

Therefore $$V_{TH} = V_{CD} = V\left(\frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4}\right)$$

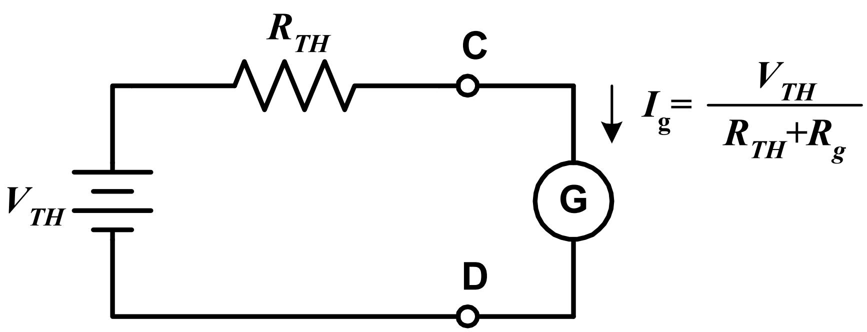
Sensitivity of Galvanometer (continued)

Thévenin Resistance (R_{TH})



$$R_{TH} = R_1 \,//\, R_3 + R_2 \,//\, R_4$$

Completed Circuit

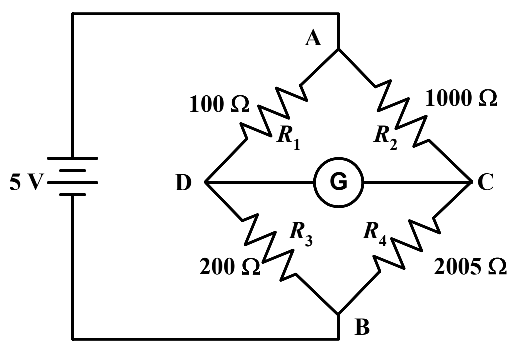


$$I_g = \frac{V_{TH}}{R_{TH} + R_g}$$

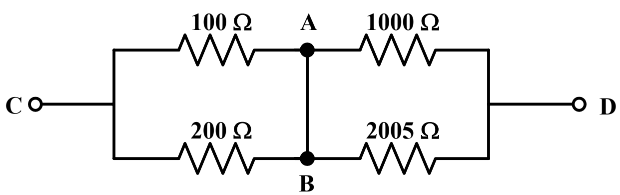
where I_g = the galvanometer current
R_g = the galvanometer resistance

Example 1 Figure below show the schematic diagram of a Wheatstone bridge with values of the bridge elements. The battery voltage is 5 V and its internal resistance negligible. The galvanometer has a current sensitivity of 10 mm/μA and an internal resistance of 100 Ω. Calculate the deflection of the galvanometer caused by the 5-Ω unbalance in arm *BC*

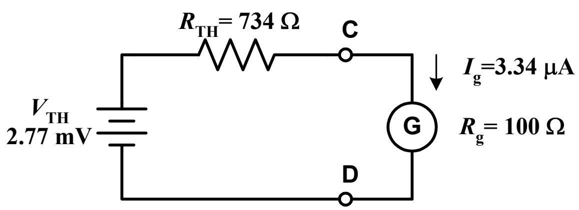
SOLUTION The bridge circuit is in the small unbalance condition since the value of resistance in arm BC is 2,005 Ω.



(a)



(b)



(c)

Thévenin Voltage (V_{TH})

$$V_{TH} = V_{AD} - V_{AC} = 5\text{ V} \times \left(\frac{100}{100+200} - \frac{1000}{1000+2005} \right)$$

$$\approx 2.77\text{ mV}$$

Thévenin Resistance (R_{TH})

$$R_{TH} = 100 // 200 + 1000 // 2005 = 734\ \Omega$$

The galvanometer current

$$I_g = \frac{V_{TH}}{R_{TH} + R_g} = \frac{2.77\text{ mV}}{734\ \Omega + 100\ \Omega} = 3.32\ \mu\text{A}$$

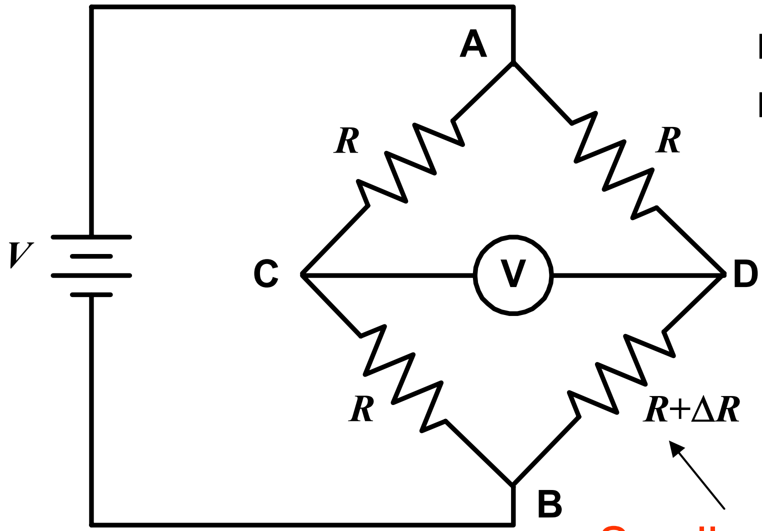
Galvanometer deflection

$$d = 3.32\ \mu\text{A} \times \frac{10\text{ mm}}{\mu\text{A}} = 33.2\text{ mm}$$

Example 2 The galvanometer in the previous example is replaced by one with an internal resistance of 500 Ω and a current sensitivity of 1mm/μA. Assuming that a deflection of 1 mm can be observed on the galvanometer scale, determine if this new galvanometer is capable of detecting the 5-Ω unbalance in arm *BC*

Example 3 If all resistances in the Example 1 increase by 10 times, and we use the galvanometer in the Example 2. Assuming that a deflection of 1 mm can be observed on the galvanometer scale, determine if this new setting can be detected (the 50-Ω unbalance in arm *BC*)

Deflection Method



Consider a bridge circuit which have identical resistors, R in three arms, and the last arm has the resistance of $R + \Delta R$. if $\Delta R/R$ >>1

Thévenin Voltage (V_{TH})

$$V_{TH} = V_{CD} = V \frac{\Delta R / R}{4 + 2\Delta R / R}$$

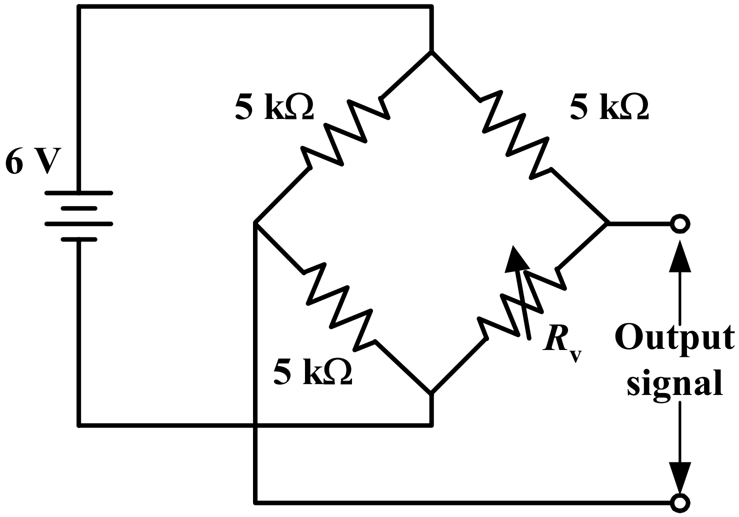
Thévenin Resistance (R_{TH})

$$R_{TH} \approx R$$

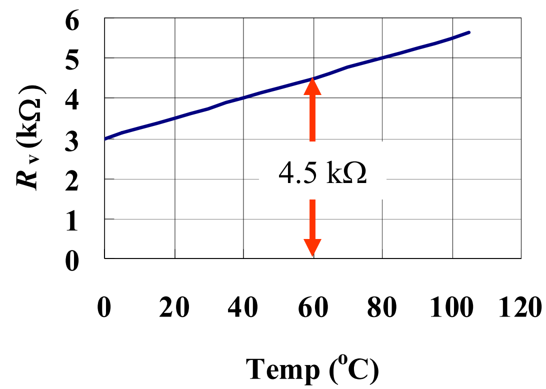
In an unbalanced condition, the magnitude of the current or voltage drop for the meter or galvanometer portion of a bridge circuit is a direct indication of the change in resistance in one arm.

This kind of bridge circuit can be found in sensor applications, where the resistance in one arm is sensitive to a physical quantity such as pressure, temperature, strain etc.

Example Circuit in Figure (a) below consists of a resistor R_v which is sensitive to the temperature change. The plot of R VS $Temp$. is also shown in Figure (b). Find (a) the temperature at which the bridge is balance and (b) The output signal at Temperature of 60^{o}C.

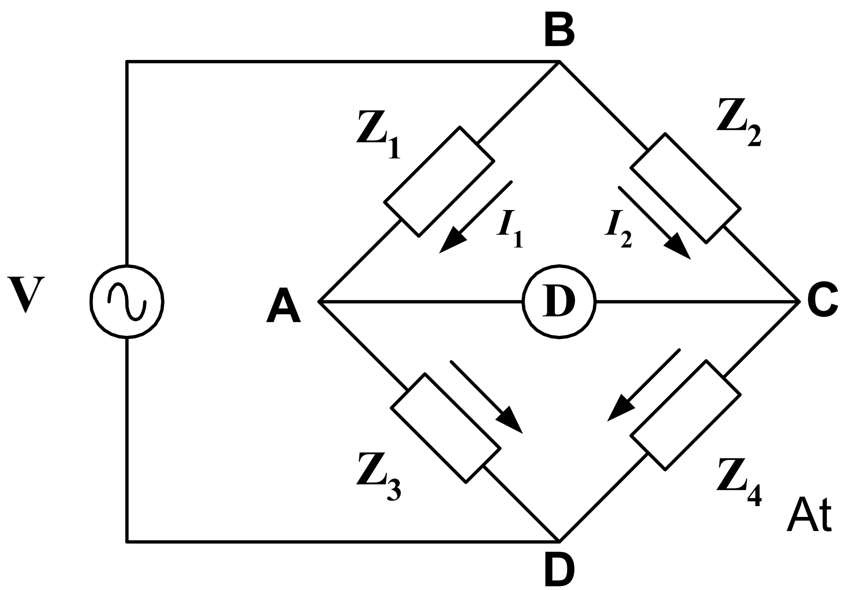


(a)



(b)

AC Bridge: Balance Condition



- all four arms are considered as impedance (frequency dependent components)
- The detector is an ac responding device: headphone, ac meter
- Source: an ac voltage at desired frequency

$\mathbf{Z_1}$, $\mathbf{Z_2}$, $\mathbf{Z_3}$ and $\mathbf{Z_4}$ are the impedance of bridge arms

At balance point: $\mathbf{E_{BA}} = \mathbf{E_{BC}}$ or $\mathbf{I_1 Z_1} = \mathbf{I_2 Z_2}$

$$\mathbf{I_1} = \frac{\mathbf{V}}{\mathbf{Z_1} + \mathbf{Z_3}} \text{ and } \mathbf{I_2} = \frac{\mathbf{V}}{\mathbf{Z_2} + \mathbf{Z_4}}$$

General Form of the ac Bridge

Complex Form: $\mathbf{Z_1 Z_4} = \mathbf{Z_2 Z_3}$

Polar Form:

$$Z_1 Z_4 (\angle\theta_1 + \angle\theta_4) = Z_2 Z_3 (\angle\theta_2 + \angle\theta_3)$$

Magnitude balance: $Z_1 Z_4 = Z_2 Z_3$

Phase balance: $\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$

Example The impedance of the basic ac bridge are given as follows:

$\mathbf{Z}_1 = 100\ \Omega\ \angle 80^{\circ}$ (inductive impedance)

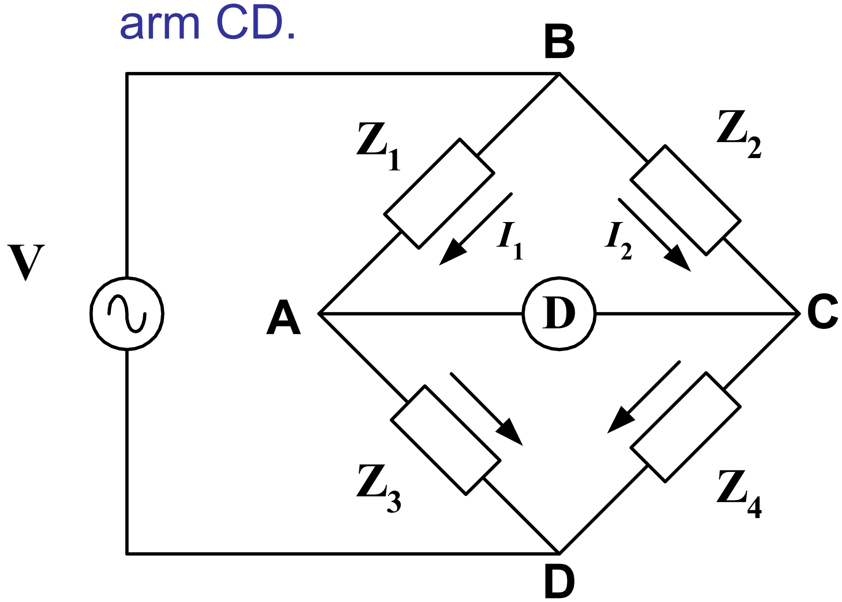
$\mathbf{Z}_2 = 250\ \Omega$ (pure resistance)

$\mathbf{Z}_3 = 400\ \angle 30^{\circ}\Omega$ (inductive impedance)

$\mathbf{Z}_4 =$ unknown

Determine the constants of the unknown arm.

Example an ac bridge is in balance with the following constants: arm AB, R = 200 Ω in series with L = 15.9 mH R; arm BC, R = 300 Ω in series with C = 0.265 μF; arm CD, unknown; arm DA, = 450 Ω. The oscillator frequency is 1 kHz. Find the constants of arm CD.



SOLUTION

$$\mathbf{Z}_1 = R + j\omega L = 200 + j100\ \Omega$$

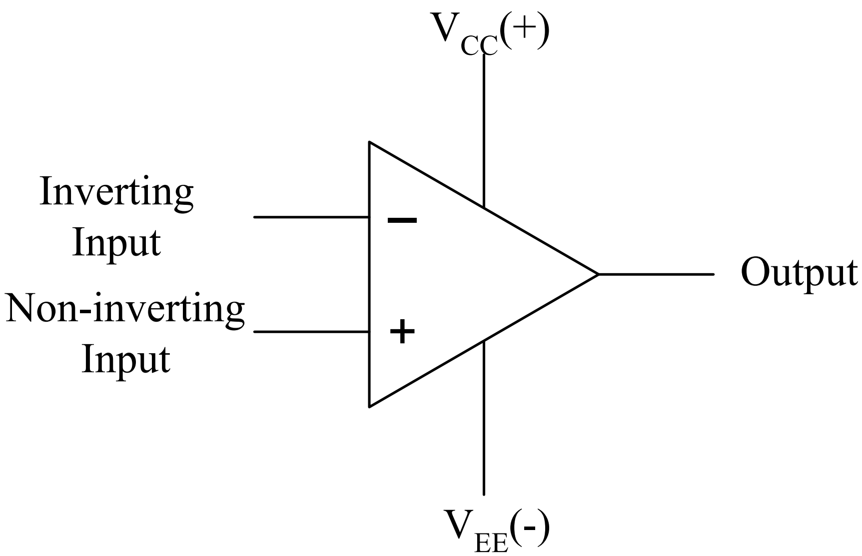
$$\mathbf{Z}_2 = R + 1/j\omega C = 300 - j600\ \Omega$$

$$\mathbf{Z}_3 = R = 450\ \Omega$$

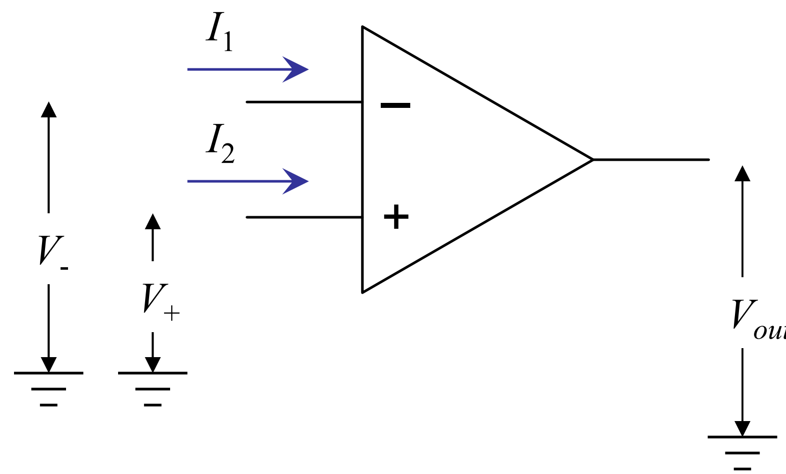
$$\mathbf{Z}_4 = \text{unknown}$$

The general equation for bridge balance states that $\mathbf{Z}_1\mathbf{Z}_4 = \mathbf{Z}_2\mathbf{Z}_3$

Operational Amplifier: Op Amp



(a) Electrical Symbol for the op amp

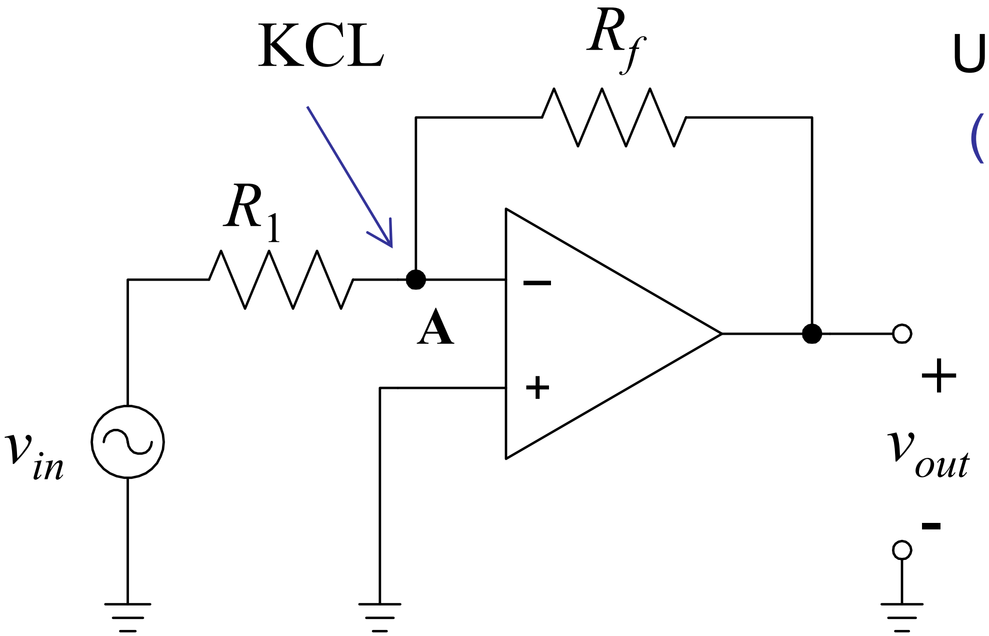


(b) Minimum connections to an op amp

Ideal Op Amp Rules:
1. No current flows in to either input terminal
2. There is no voltage difference between the two input terminals

Rule 1: $I_1 = I_2 = 0$; R+/- = ∞
Rule 2: $V+ = V-$; **Virtually shorted**

Inverting Amplifier



Use KCL at point A and apply Rule 1:

(no current flows into the inverting input)

$$\frac{v_A - v_{in}}{R_1} + \frac{v_A - v_{out}}{R_f} = 0$$

Rearrange

$$v_A\left(\frac{1}{R_1} + \frac{1}{R_f}\right) - \left(\frac{v_{in}}{R_1} + \frac{v_{out}}{R_f}\right) = 0$$

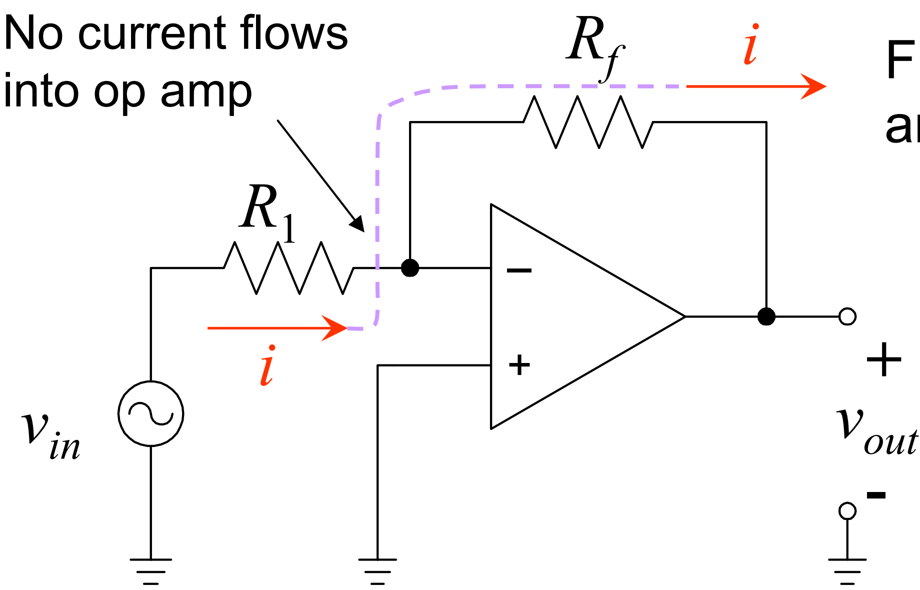
Apply Rule 2: (no voltage difference between inverting and non-inverting inputs)

Since V+ at zero volts, therefore V- is also at zero volts too.

$$v_A = 0$$

$$\frac{v_{in}}{R_1} + \frac{v_{out}}{R_f} = 0 \quad \Longrightarrow \quad \frac{v_{out}}{v_{in}} = -\frac{R_f}{R_1}$$

Inverting Amplifier: another approach



From Rule 2: we know that $V- = V+ = 0$, and therefore

$$-v_{in} + iR_1 - \overset{0}{V^-} = 0 \Rightarrow i = \frac{v_{in}}{R_1}$$

Since there is no current into op amp (Rule 1)

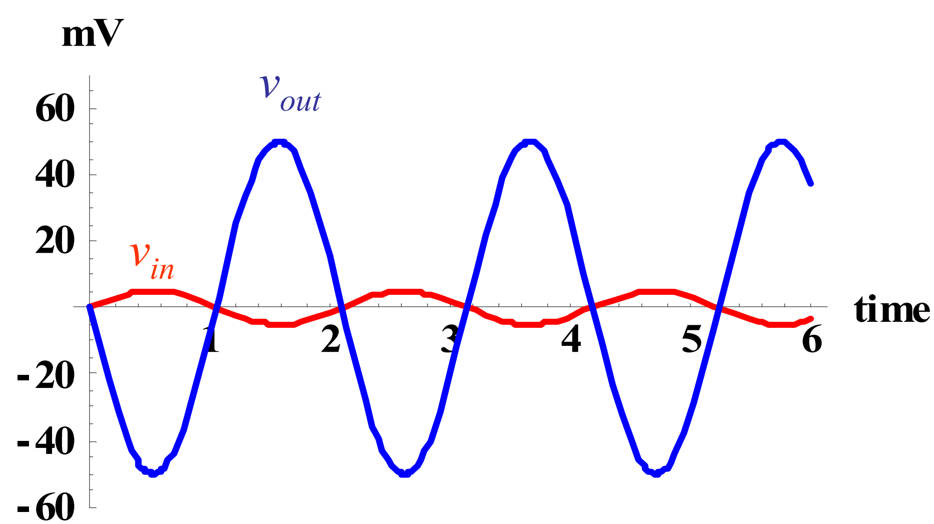
$$-\overset{0}{V^-} + iR_f + v_{out} = 0 \Rightarrow v_{out} = -iR_f$$

Combine the results, we get

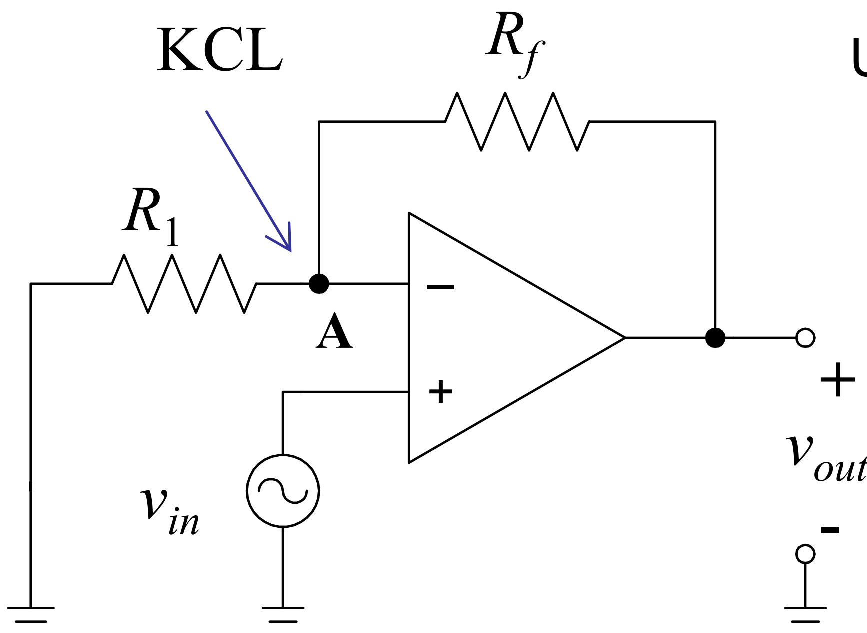
$$\frac{v_{out}}{v_{in}} = -\frac{R_f}{R_1}$$

Given $v_{in} = 5\sin 3t$, R_1=4.7 kΩ and R_f=47 kΩ

$$v_{out} = -10v_{in} = -50 \sin 3t \quad \text{mV}$$



Non-inverting Amplifier



Use KCL at point A and apply Rule 1:

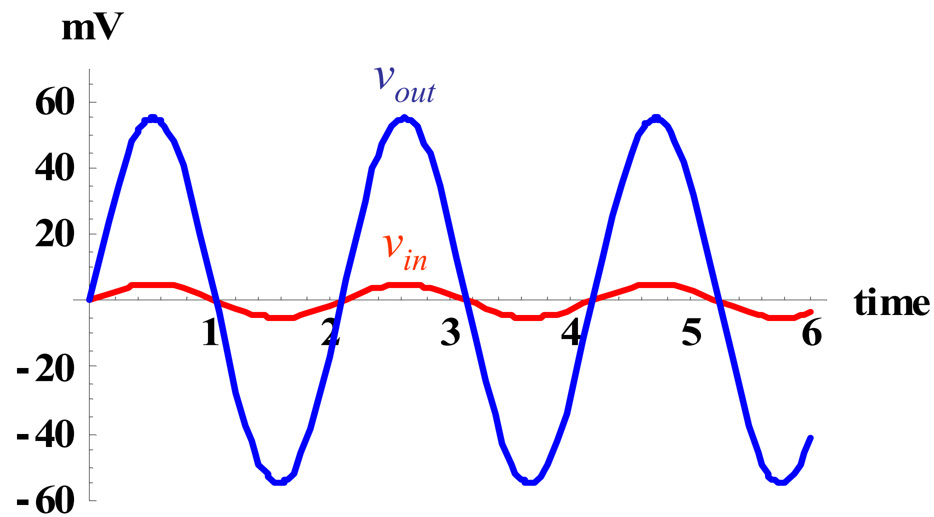
$$\frac{v_A}{R_1} + \frac{v_A - v_{out}}{R_f} = 0$$

Apply Rule 2: $v_{in} = v_A$

$$\frac{v_{out}}{v_{in}} = 1 + \frac{R_f}{R_1}$$

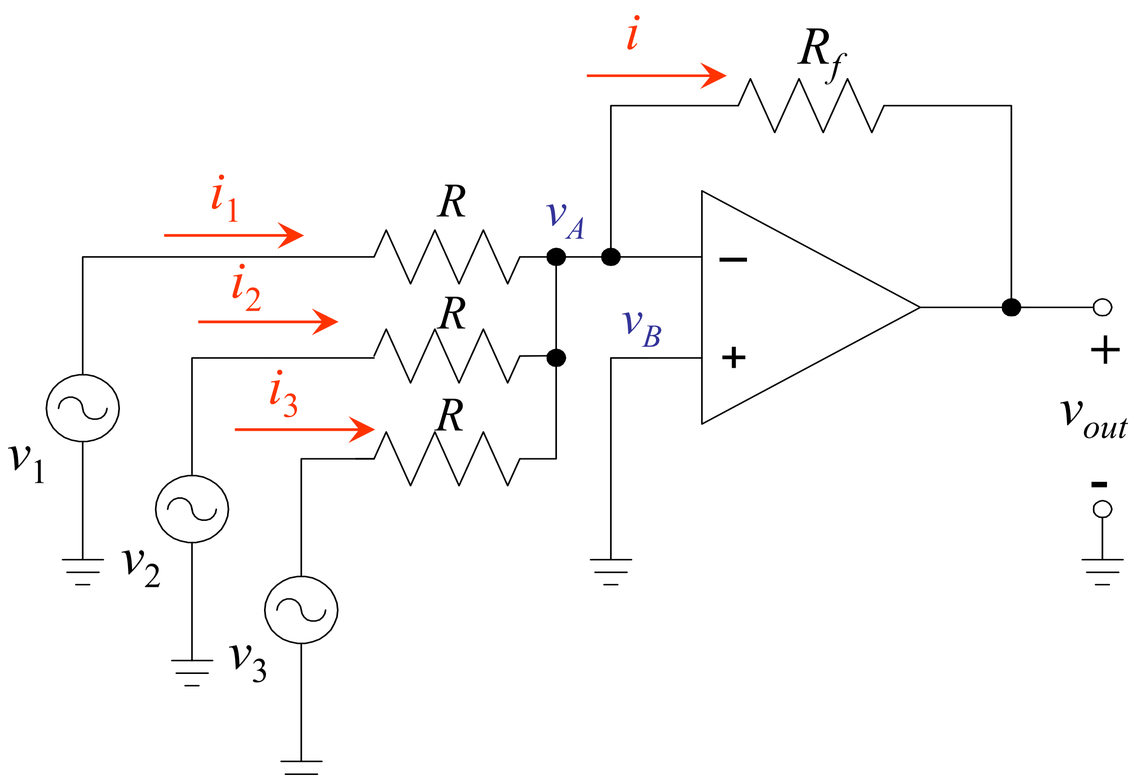
Given $v_{in} = 5\sin 3t$, R_1=4.7 kΩ and R_f=47 kΩ

$$v_{out} = 11 v_{\text{in}} = 55 \sin 3t \qquad \text{mV}$$



Summing Amplifier: Mathematic Operation

$$i = i_1 + i_2 + i_3$$



Use KCL and apply Rule 1:

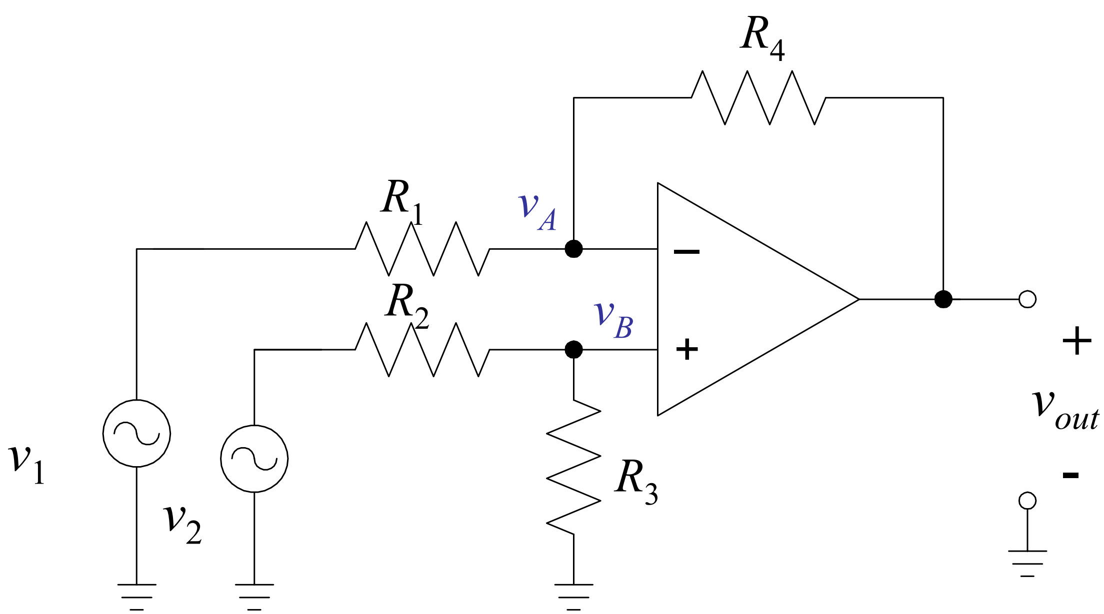
$$\frac{v_A - v_1}{R} + \frac{v_A - v_2}{R} + \frac{v_A - v_3}{R} + \frac{v_A - v_{out}}{R_f} = 0$$

Since v_A = 0 (Rule 2)

$$v_{out} = -\frac{R_f}{R}\left(v_1 + v_2 + v_3\right)$$

Sum of v_1, v_2 and v_3

Difference Amplifier: Mathematic Operation



Use KCL and apply Rule 1:

$$\frac{v_A - v_1}{R_1} + \frac{v_A - v_{out}}{R_4} = 0 \qquad (1)$$

Since v_A = v_B (Rule 2) and

$$v_A = v_B = \left(\frac{R_3}{R_2 + R_3}\right) v_2 \qquad (2)$$

Substitute eq. (2) into eq. (1), we get

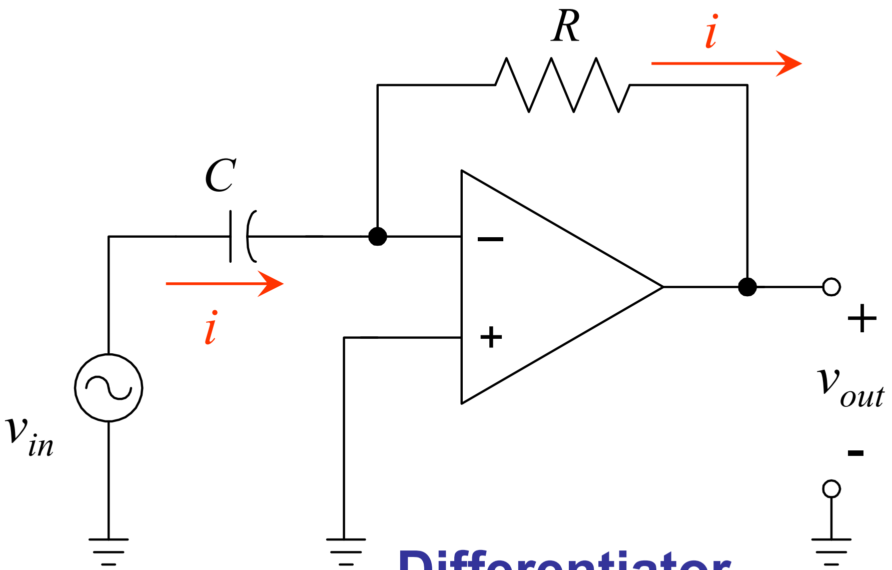
$$\frac{v_{out}}{R_4} = \left(\frac{R_1 + R_4}{R_1 R_4}\right)\left(\frac{R_3}{R_2 + R_3}\right) v_2 - \frac{v_1}{R_1}$$

If $R_1 = R_2 = R$ and $R_3 = R_4 = R_f$

$$v_{out} = \frac{R_f}{R}\left(v_2 - v_1\right)$$

Difference of v_1 and v_2

Differentiator and Integrator: Mathematic Operation

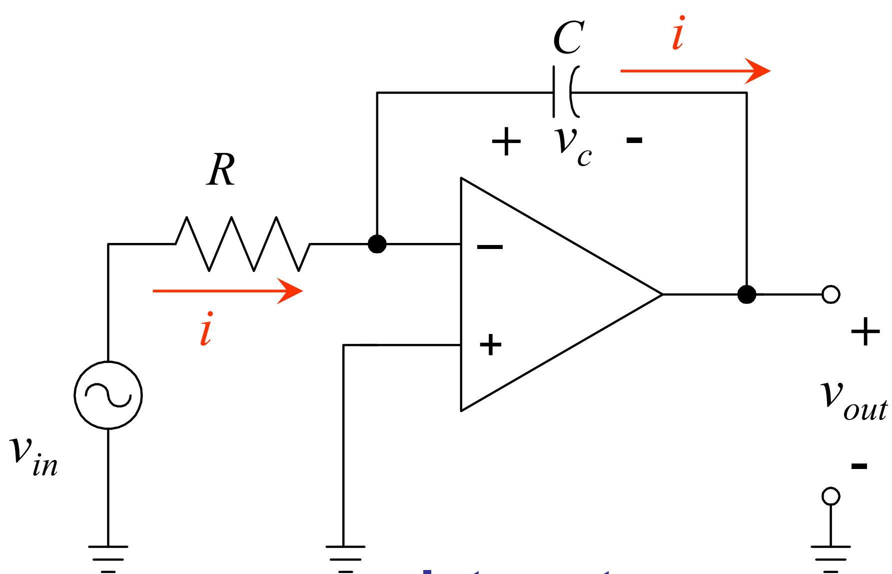


Differentiator

$$v_{out} = -iR$$

But $i = C\dfrac{dv_C}{dt}$ and $v_{in} = v_C$

$$v_{out} = -RC\frac{dv_{in}}{dt}$$



Integrator

$$v_{out} = -v_C$$

But $v_C(t) = \dfrac{1}{C}\displaystyle\int_0^t i\,dt + v_C(0)$ and $v_{in} = iR$

$$v_{out} = -\frac{1}{RC}\int_0^t v_{in}\,dt + v_C(0)$$