Linear combination of Normal Random Variables

Linear Function of a Normal Random Variable

If \( X \sim N(\mu, \sigma^2) \) and \( a \) and \( b \) are constants, then

\[
Y = aX + b \sim N(a\mu + b, a^2 \sigma^2)
\]

Linear Combinations of Independent Normal Random Variable

If \( X_i \sim N(\mu_i, \sigma_i^2) \), \( 1 \leq i \leq n \), are independent variables and if \( a_i, 1 \leq i \leq n \) and \( b \) are constants, then

\[
Y = a_1X_1 + \ldots + a_nX_n + b \sim N(\mu, \sigma^2)
\]

Where

\[
\mu = a_1\mu_1 + \ldots + a_n\mu_n + b
\]

and

\[
\sigma^2 = a_1^2\sigma_1^2 + \ldots + a_n^2\sigma_n^2
\]

Average Independent Normal Random Variable

If \( X_i \sim N(\mu, \sigma^2) \), \( 1 \leq i \leq n \), are independent variables, then their average \( \overline{X} \) is distributed

\[
\overline{X} \sim N(\mu, \frac{\sigma^2}{n})
\]

The Central Limit Theorem

If \( X_1, \ldots, X_n \), is a sequence of independent identically distributed random variables with a mean and a variance, then the distribution of their average \( \overline{X} \) can be approximated by a

\[
N(\mu, \frac{\sigma^2}{n})
\]

distribution. Similarly, the distribution of the sum \( X_1 + \ldots + X_n \) can be approximated by a \( N(n\mu, n\sigma^2) \) distribution.

The Chi-Square Distribution

A Chi-square random variable with \( v \) degrees of freedom, \( X \) can be generated as

\[
X = X_1^2 + \ldots + X_v^2
\]

Where \( X_i \) are independent standard normal random variables. A chi-square distribution with \( v \) degrees of freedom is a gamma distribution with parameter values and , it has an expectation of \( v \) and a variance of \( 2v \)

The t-Distribution

A t-Distribution with \( v \) degrees of freedom, \( X \) is defined to be

\[
t_v \sim \frac{N(0,1)}{\sqrt{\chi^2 / v}}
\]
Where $N(0,1)$ and $\chi^2_v$ random variables are independently distributed. The $t$-distribution has a shape similar to a standard normal distribution but is a little flatter. As $v \to \infty$, the $t$-distribution tends to a standard normal distribution.

**Sample Mean**

If $X_1,\ldots,X_n$, are observations from a population with a mean $\mu$ and a variance $\sigma^2$, then the central limit theorem indicates that the sample mean $\hat{\mu} = \bar{X}$ has the approximate distribution

$$
\hat{\mu} = \bar{X} \sim N(\mu, \frac{\sigma^2}{n})
$$

**Sample Variance**

If $X_1,\ldots,X_n$, are normally distribution with a mean $\mu$ and a variance $\sigma^2$, then the sample variance $S^2$

$$
S^2 \sim \sigma^2 \frac{\chi^2_{n-1}}{n-1}
$$

**t-Statistic**

If $X_1,\ldots,X_n$, are normally distribution with a mean $\mu$ then

$$
\frac{\sqrt{n}(\bar{X} - \mu)}{S} \sim t_{n-1}
$$

This result is very important since in practice an experimenter knows the value of $n$ and the observed sample mean $\bar{X}$ and sample variance $S^2$, and so knows everything in the quantity $\frac{\sqrt{n}(\bar{X} - \mu)}{S}$ except for $\mu$. This allows the experimenter to make useful inferences about $\mu$. 