Linear combination of Normal Random Variables

Linear Function of a Normal Random Variable

If $X \sim N(\mu, \sigma^2)$ and a and b are constants, then

$Y = aX + b \sim N(a\mu + b, a^2\sigma^2)$

Linear Combinations of Independent Normal Random Variable

If $X_i \sim N(\mu_i, \sigma_i^2)$, $1 \le i \le n$, are independent variables and if a_i, $1 \le i \le n$ and b are constants, then

$Y = a_1 X_1 ... + a_n X_n + b \sim N(\mu, \sigma^2)$

Where

$\mu = a_1\mu_1 ... + a_n\mu_n + b$

and

$\sigma^2 = a_1^2\sigma_1^2 + ... + a_n^2\sigma_n^2$

Average Independent Normal Random Variable

If $X_i \sim N(\mu, \sigma^2)$, $1 \le i \le n$, are independent variables, then their average $\overline{X}$ is distributed

$\overline{X} \sim N(\mu, \frac{\sigma^2}{n})$

The Central Limit Theorem

If $X_1, ..., X_n$, is a sequence of independent identically distributed random variables with a mean and a variance, then the distribution of their average $\overline{X}$ can be approximated by a

$N(\mu, \frac{\sigma^2}{n})$

distribution, Similarly, the distribution of the sum $X_1 + ... + X_n$ can be approximated by a

$N(n\mu, n\sigma^2)$

distribution.

The Chi-Square Distribution

A Chi-square random variable with v degrees of freedom, X can be generated as

$X = X_1^2 + ... + X_v^2$

Where X_i are independent standard normal random variables. A chi-square distribution with v degrees of freedom is a gamma distribution with parameter values and , it has an expectation of v and a variance of $2v$

The *t*-Distribution

A *t*-Distribution with v degrees of freedom, X is defined to be

$t_v \sim \frac{N(0,1}{\sqrt{\chi^2 / v}}$

Where $N(0,1)$ and χ_v^2 random variables are independently distributed. The *t*-distribution has a shape similar to a standard normal distribution but is a little flatter. As $v \to \infty$, the *t*-distribution tends to a standard normal distribution.

Sample Mean

If $X_1, ..., X_n$, are observations from a population with a mean μ and a variance σ^2 then the central limit theorem indicates that the sample mean $\hat{\mu} = \overline{X}$ has the approximate distribution

$$\hat{\mu} = \overline{X} \sim N(\mu, \frac{\sigma^2}{n})$$

Sample Variance

If $X_1, ..., X_n$, are normally distribution with a mean μ and a variance σ^2 then the sample variance S^2

$$S^2 \sim \sigma^2 \frac{\chi_{n-1}^2}{n-1}$$

***t*-Statistic**

If $X_1, ..., X_n$, are normally distribution with a mean μ then

$$\frac{\sqrt{n}\left(\overline{X} - \mu\right)}{S} \sim t_{n-1}$$

This result is very important since in practice an experimenter knows the value of n and the observed sample mean $\overline{X}$ and sample variance S^2, and so knows everything in the quantity

$$\frac{\sqrt{n}\left(\overline{X} - \mu\right)}{S}$$

except for μ. This allows the experimenter to make useful inferences about μ