2102311 Electrical Measurement and Instruments (Part II)

- Bridge Circuits (DC and AC)
- Electronic Instruments (Analog & Digital)
- Signal Generators
- Frequency and Time Interval Measurements
- Introduction to Transducers

Textbook:
# Resistor Types

## Importance parameters
- Value
- Tolerance
- Power rating
- Temperature coefficient

<table>
<thead>
<tr>
<th>Type</th>
<th>Values (Ω)</th>
<th>Power rating (W)</th>
<th>Tolerance (%)</th>
<th>Temperature coefficient (ppm/°C)</th>
<th>picture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire wound (power)</td>
<td>10m~3k</td>
<td>3~1k</td>
<td>±1~±10</td>
<td>±30~±300</td>
<td><img src="image1.png" alt="Picture" /></td>
</tr>
<tr>
<td>Wire wound (precision)</td>
<td>10m~1M</td>
<td>0.1~1</td>
<td>±0.005~±1</td>
<td>±3~±30</td>
<td><img src="image2.png" alt="Picture" /></td>
</tr>
<tr>
<td>Carbon film</td>
<td>1~1M</td>
<td>0.1~3</td>
<td>±2~±10</td>
<td>±100~±200</td>
<td><img src="image3.png" alt="Picture" /></td>
</tr>
<tr>
<td>Metal film</td>
<td>100m~1M</td>
<td>0.1~3</td>
<td>±0.5~±5</td>
<td>±10~±200</td>
<td><img src="image4.png" alt="Picture" /></td>
</tr>
<tr>
<td>Metal film (precision)</td>
<td>10m~100k</td>
<td>0.1~1</td>
<td>±0.05~±5</td>
<td>±0.4~±10</td>
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<tr>
<td>Metal oxide film</td>
<td>100m~100k</td>
<td>1~10</td>
<td>±2~±10</td>
<td>±200~±500</td>
<td><img src="image6.png" alt="Picture" /></td>
</tr>
</tbody>
</table>

Data: Transistor technology (10/2000)
## Resistor Values

### Color codes

<table>
<thead>
<tr>
<th>Color</th>
<th>Digit</th>
<th>Multiplier</th>
<th>Tolerance (%)</th>
<th>Temperature coefficient (ppm/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver</td>
<td>-</td>
<td>10^{-2}</td>
<td>±10</td>
<td>K</td>
</tr>
<tr>
<td>Gold</td>
<td>-</td>
<td>10^{-1}</td>
<td>±5</td>
<td>J</td>
</tr>
<tr>
<td>Black</td>
<td>0</td>
<td>10^{0}</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Brown</td>
<td>1</td>
<td>10^{1}</td>
<td>±1</td>
<td>F</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>10^{2}</td>
<td>±2</td>
<td>G</td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
<td>10^{3}</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
<td>10^{4}</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
<td>10^{5}</td>
<td>±0.5</td>
<td>D</td>
</tr>
<tr>
<td>Blue</td>
<td>6</td>
<td>10^{6}</td>
<td>±0.25</td>
<td>C</td>
</tr>
<tr>
<td>Violet</td>
<td>7</td>
<td>10^{7}</td>
<td>±0.1</td>
<td>B</td>
</tr>
<tr>
<td>Gray</td>
<td>8</td>
<td>10^{8}</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>White</td>
<td>9</td>
<td>10^{9}</td>
<td>±20</td>
<td>M</td>
</tr>
</tbody>
</table>

### Alphanumeric

- **R, K, M, G, and T** = x10^0, x10^3, x10^6, x10^9, and x10^{12}

### Ex.

- **Ex. 6M8** = 6.8 x 10^6 Ω
- **59P04** = 59.04 Ω

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Data: Transistor technology (10/2000)
Resistor Values

\[ R = x \pm \% \Delta x \]

Tolerance
Nominal value

Ex. 1 kΩ ± 10% \( \equiv \) 900-1100 Ω

For 10% resistor
10, 12, 15, 18, …

For 10% resistor: \( E = 12 \)

\[ n = 0; R = 1.00000… \]
\[ n = 1; R = 1.21152… \]
\[ n = 2; R = 1.46779… \]
\[ n = 3; R = 1.77827… \]

Commonly available resistance for a fixed resistor

<table>
<thead>
<tr>
<th>±1%</th>
<th>±2%</th>
<th>±5%</th>
<th>±10%</th>
<th>±1%</th>
<th>±2%</th>
<th>±5%</th>
<th>±10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td>10</td>
<td>10</td>
<td>316</td>
<td>316</td>
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<td>102</td>
<td>10</td>
<td>10</td>
<td>318</td>
<td>318</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
<td>…</td>
</tr>
<tr>
<td>300</td>
<td>300</td>
<td>30</td>
<td>30</td>
<td>953</td>
<td>953</td>
<td>95</td>
<td>95</td>
</tr>
<tr>
<td>301</td>
<td>301</td>
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<td>30</td>
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<td>954</td>
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<td>30</td>
<td>30</td>
<td>955</td>
<td>955</td>
<td>95</td>
<td>95</td>
</tr>
</tbody>
</table>

where \( E = 6, 12, 24, 96 \)
for 20, 10, 5, 1% tolerance
\[ n = 0, 1, 2, 3, … \]
Resistance Measurement Techniques

- Bridge circuit
- Voltmeter-ammeter
- Substitution
- Ohmmeter

Voltmeter-ammeter

Substitution

Decade resistance box substituted in place of the unknown
Voltmeter-ammeter method

Pro and con:
- Simple and theoretical oriented
- Requires two meter and calculations
- Subject to error: Voltage drop in ammeter (Fig. (a))
  Current in voltmeter (Fig. (b))

![Fig. (a)](image1)

![Fig. (b)](image2)

Measured $R_x$:

- **Fig. (a)**
  $$R_{\text{meas}} = \frac{V}{I} = \frac{V_x + V_A}{I} = R_x + \frac{V_A}{I}$$

  If $V_x >> V_A$

  $$R_{\text{meas}} \approx R_x$$

Therefore this circuit is suitable for measure large resistance

- **Fig. (b)**
  $$R_{\text{meas}} = \frac{V}{I} = \frac{V}{I_x + I_V} = \frac{R_x}{1 + I_V / I_x}$$

  If $I_x >> I_V$

  $$R_{\text{meas}} \approx R_x$$

Therefore this circuit is suitable for measure small resistance
Ohmmeter

- Voltmeter-ammeter method is rarely used in practical applications (mostly used in Laboratory)
- Ohmmeter uses only one meter by keeping one parameter constant

**Example: series ohmmeter**

Basic series ohmmeter consisting of a PMMC and a series-connected standard resistor ($R_1$). When the ohmmeter terminals are shorted ($R_x = 0$) meter full scale defection occurs. At half scale defection $R_x = R_1 + R_m$, and at zero defection the terminals are open-circuited.
Bridge Circuit

Bridge Circuit is a null method, operates on the principle of comparison. That is a known (standard) value is adjusted until it is equal to the unknown value.

DC Bridge (Resistance)
- Wheatstone Bridge
- Kelvin Bridge
- Megaohm Bridge

AC Bridge
- Maxwell Bridge
- Hay Bridge
- Owen Bridge
- Etc.
- Schering Bridge
- Wien Bridge

Inductance
- Capacitance
- Frequency
Wheatstone Bridge and Balance Condition

Suitable for moderate resistance values: $1 \, \Omega$ to $10 \, M\Omega$

Balance condition:
No potential difference across the galvanometer (there is no current through the galvanometer)

Under this condition: $V_{AD} = V_{AB}$

$I_1R_1 = I_2R_2$

And also $V_{DC} = V_{BC}$

$I_3R_3 = I_4R_4$

where $I_1$, $I_2$, $I_3$, and $I_4$ are current in resistance arms respectively, since $I_1 = I_3$ and $I_2 = I_4$

$\frac{R_1}{R_3} = \frac{R_2}{R_4}$ or $R_x = R_4 = R_3 \frac{R_2}{R_1}$
Example

(a) Equal resistance

(b) Proportional resistance

(c) Proportional resistance

(d) 2-Volt unbalance
Measurement Errors

1. Limiting error of the known resistors

Using 1st order approximation:

\[ R_x = \left( R_3 \pm \Delta R_3 \right) \left( \frac{R_2 \pm \Delta R_2}{R_1 \pm \Delta R_1} \right) \]

\[ R_x = R_3 \frac{R_2}{R_1} \left( 1 \pm \frac{\Delta R_1}{R_1} \pm \frac{\Delta R_2}{R_2} \pm \frac{\Delta R_3}{R_3} \right) \]

2. Insufficient sensitivity of Detector

3. Changes in resistance of the bridge arms due to the heating effect \((I^2R)\) or temperatures

4. Thermal emf or contact potential in the bridge circuit

5. Error due to the lead connection

3, 4 and 5 play the important role in the measurement of low value resistance
Example  In the Wheatstone bridge circuit, $R_3$ is a decade resistance with a specified in accuracy $\pm 0.2\%$ and $R_1$ and $R_2 = 500 \ \Omega \pm 0.1\%$. If the value of $R_3$ at the null position is $520.4 \ \Omega$, determine the possible minimum and maximum value of $R_x$.

SOLUTION  Apply the error equation

$$R_x = R_3 \frac{R_2}{R_1} \left(1 \pm \frac{\Delta R_1}{R_1} \pm \frac{\Delta R_2}{R_2} \pm \frac{\Delta R_3}{R_3}\right)$$

$$R_x = \frac{520.4 \times 500}{500} \left(1 \pm \frac{0.1}{100} \pm \frac{0.1}{100} \pm \frac{0.2}{100}\right) = 520.4(1 \pm 0.004) = 520.4 \pm 0.4\%$$

Therefore the possible values of $R_3$ are 518.32 to 522.48 $\Omega$.

Example  A Wheatstone bridge has a ratio arm of $1/100 \ (R_2/R_1)$. At first balance, $R_3$ is adjusted to 1000.3 $\Omega$. The value of $R_x$ is then changed by the temperature change, the new value of $R_3$ to achieve the balance condition again is 1002.1 $\Omega$. Find the change of $R_x$ due to the temperature change.

SOLUTION  At first balance: $R_x$ old $= R_3 \frac{R_2}{R_1} = 1000.3 \times \frac{1}{100} = 10.003 \ \Omega$

After the temperature change: $R_x$ new $= R_3 \frac{R_2}{R_1} = 1002.1 \times \frac{1}{100} = 10.021 \ \Omega$

Therefore, the change of $R_x$ due to the temperature change is $0.018 \ \Omega$. 
A galvanometer is used to detect an unbalance condition in Wheatstone bridge. Its sensitivity is governed by: \textit{Current sensitivity (currents per unit deflection) and internal resistance.}

Consider a bridge circuit under a small unbalance condition, and apply circuit analysis to solve the current through galvanometer.

\section{Thévenin Equivalent Circuit}

The Thévenin Voltage ($V_{TH}$)

\[ V_{CD} = V_{AC} - V_{AD} = I_1 R_1 - I_2 R_2 \]

where

\[ I_1 = \frac{V}{R_1 + R_3} \quad \text{and} \quad I_2 = \frac{V}{R_2 + R_4} \]

Therefore

\[ V_{TH} = V_{CD} = V \left( \frac{R_1}{R_1 + R_3} - \frac{R_2}{R_2 + R_4} \right) \]
Sensitivity of Galvanometer (continued)

**Thévenin Resistance** ($R_{TH}$)

\[
R_{TH} = R_1 // (R_3 + R_2) // R_4
\]

**Completed Circuit**

\[ I_g = \frac{V_{TH}}{R_{TH} + R_g} \]

where $I_g$ = the galvanometer current

$R_g$ = the galvanometer resistance
Example 1: Figure below show the schematic diagram of a Wheatstone bridge with values of the bridge elements. The battery voltage is 5 V and its internal resistance negligible. The galvanometer has a current sensitivity of 10 mm/µA and an internal resistance of 100 Ω. Calculate the deflection of the galvanometer caused by the 5-Ω unbalance in arm BC.

**SOLUTION** The bridge circuit is in the small unbalance condition since the value of resistance in arm BC is 2,005 Ω.

![Bridge Circuit Diagram](image)

**Thévenin Voltage \( (V_{TH}) \)**

\[
V_{TH} = V_{AD} - V_{AC} = 5 \text{ V} \times \left( \frac{100}{100 + 200} - \frac{1000}{1000 + 2005} \right)
\]

\[
\approx 2.77 \text{ mV}
\]

**Thévenin Resistance \( (R_{TH}) \)**

\[
R_{TH} = 100//200 + 1000//2005 = 734 \text{ Ω}
\]

The galvanometer current

\[
I_g = \frac{V_{TH}}{R_{TH} + R_g} = \frac{2.77 \text{ mV}}{734 \text{ Ω} + 100 \text{ Ω}} = 3.32 \text{ µA}
\]

Galvanometer deflection

\[
d = 3.32 \text{ µA} \times \frac{10 \text{ mm}}{\text{µA}} = 33.2 \text{ mm}
\]
Example 2 The galvanometer in the previous example is replaced by one with an internal resistance of 500 Ω and a current sensitivity of 1 mm/µA. Assuming that a deflection of 1 mm can be observed on the galvanometer scale, determine if this new galvanometer is capable of detecting the 5-Ω unbalance in arm BC.

SOLUTION Since the bridge constants have not been changed, the equivalent circuit is again represented by a Thévenin voltage of 2.77 mV and a Thévenin resistance of 734 Ω. The new galvanometer is now connected to the output terminals, resulting a galvanometer current.

\[
I_g = \frac{V_{TH}}{R_{TH} + R_g} = \frac{2.77 \text{ mV}}{734 \Omega + 500 \Omega} = 2.24 \text{ µA}
\]

The galvanometer deflection therefore equals 2.24 µA x 1 mm/µA = 2.24 mm, indicating that this galvanometer produces a deflection that can be easily observed.
Example 3 If all resistances in the Example 1 increase by 10 times, and we use the galvanometer in the Example 2. Assuming that a deflection of 1 mm can be observed on the galvanometer scale, determine if this new setting can be detected (the 50-Ω unbalance in arm $BC$)

SOLUTION
Application of Wheatstone Bridge

Murray/Varrley Loop Short Circuit Fault (Loop Test)

- Loop test can be carried out for the location of either a ground or a short circuit fault.

Assume: earth is a good conductor

Let $R = R_1 + R_2$

At balance condition:

$$\frac{R_3}{R_4} = \frac{R_1}{R_2}$$

$$R_1 = R \left( \frac{R_3}{R_3 + R_4} \right)$$

$$R_2 = R \left( \frac{R_4}{R_3 + R_4} \right)$$

The value of $R_1$ and $R_2$ are used to calculate back into distance.
Murray/Varrley Loop Short Circuit Fault (Loop Test)

Examples of commonly used cables (Approx. R at 20°C)

<table>
<thead>
<tr>
<th>Wire dia. In mm</th>
<th>Ohms per km.</th>
<th>Meter per ohm</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.32</td>
<td>218.0</td>
<td>4.59</td>
</tr>
<tr>
<td>0.40</td>
<td>136.0</td>
<td>7.35</td>
</tr>
<tr>
<td>0.50</td>
<td>84.0</td>
<td>11.90</td>
</tr>
<tr>
<td>0.63</td>
<td>54.5</td>
<td>18.35</td>
</tr>
<tr>
<td>0.90</td>
<td>27.2</td>
<td>36.76</td>
</tr>
</tbody>
</table>

Remark: The resistance of copper increases 0.4% for 1°C rise in Temp.

Let \( R = R_1 + R_2 \) and define \( \text{Ratio} = \frac{R_4}{R_5} \)

At balance condition: \( \text{Ratio} = \frac{R_4}{R_5} = \frac{R_1}{R_2 + R_3} \)

\[
R_1 = \frac{\text{Ratio}}{\text{Ratio} + 1} R + R_3
\]

\[
R_2 = \frac{R - \text{Ratio}R_3}{\text{Ratio} + 1}
\]

Varley Loop Test
Example Murray loop test is used to locate ground fault in a telephone system. The total resistance, $R = R_1 + R_2$ is measured by Wheatstone bridge, and its value is 300 Ω. The conditions for Murray loop test are as follows:

$R_3 = 1000 \, \Omega$ and $R_4 = 500 \, \Omega$

Find the location of the fault in meter, if the length per Ohm is 36.67 m.

\[
R_1 = R \left( \frac{R_3}{R_3 + R_4} \right) = 300 \times \frac{1000}{1000 + 500} = 200 \, \Omega
\]

\[
R_2 = R \left( \frac{R_4}{R_3 + R_4} \right) = 300 \times \frac{500}{1000 + 500} = 100 \, \Omega
\]

Therefore, the location from the measurement point is $100 \, \Omega \times 36.67 \, m/\Omega = 3667 \, m$
Application of Wheatstone Bridge

Unbalance bridge

Consider a bridge circuit which have identical resistors, $R$ in three arms, and the last arm has the resistance of $R + \Delta R$. If $\Delta R/R \ll 1$

Small unbalance occur by the external environment

Thévenin Voltage ($V_{TH}$)

$$V_{TH} = V_{CD} \approx V \frac{\Delta R}{4R}$$

Thévenin Resistance ($R_{TH}$)

$$R_{TH} \approx R$$

This kind of bridge circuit can be found in sensor applications, where the resistance in one arm is sensitive to a physical quantity such as pressure, temperature, strain etc.
Example Circuit in Figure (a) below consists of a resistor $R_v$ which is sensitive to the temperature change. The plot of $R$ vs. $Temp.$ is also shown in Figure (b). Find (a) the temperature at which the bridge is balance and (b) The output signal at Temperature of 60° C.

![Circuit Diagram](image)

**SOLUTION** (a) at bridge balance, we have

$$R_v = \frac{R_3 \times R_2}{R_1} = \frac{5 \, \text{k\Omega} \times 5 \, \text{k\Omega}}{5 \, \text{k\Omega}} = 5 \, \text{k\Omega}$$

The value of $R_v = 5 \, \text{k\Omega}$ corresponding to the temperature of 80° C in the given plot.

(b) at temperature of 60° C, $R_v$ is read as 4.5 k\Omega, thus $\Delta R = 5 - 4.5 = 0.5 \, \text{k\Omega}$. We will use Thévenin equivalent circuit to solve the above problem.

$$V_{TH} = V \frac{\Delta R}{4R} = 6 \, \text{V} \times \frac{0.5 \, \text{k\Omega}}{4 \times 5 \, \text{k\Omega}} = 0.15 \, \text{V}$$

It should be noted that $\Delta R = 0.5 \, \text{k\Omega}$ in the problem does not satisfy the assumption $\Delta R / R \ll 1$, the exact calculation gives $V_{TH} = 0.158 \, \text{V}$. However, the above calculation still gives an acceptable solution.
Low resistance Bridge: \( R_x < 1 \ \Omega \)

**Effect of connecting lead**

The effects of the connecting lead and the connecting terminals are prominent when the value of \( R_x \) decreases to a few Ohms.

\[
R_y = \text{the resistance of the connecting lead from } R_3 \text{ to } R_x
\]

At point \( m \): \( R_y \) is added to the unknown \( R_x \), resulting in too high and indication of \( R_x \).

At point \( n \): \( R_y \) is added to \( R_3 \), therefore the measurement of \( R_x \) will be lower than it should be.

At point \( p \):

\[
R_x + R_{np} = \left( R_3 + R_{mp} \right) \frac{R_1}{R_2}
\]

rearrange

\[
R_x = R_3 \frac{R_1}{R_2} + R_{mp} \frac{R_1}{R_2} - R_{np}
\]

Where \( R_{mp} \) and \( R_{np} \) are the lead resistance from \( m \) to \( p \) and \( n \) to \( p \), respectively.

The effect of the connecting lead will be canceled out, if the sum of 2\textsuperscript{nd} and 3\textsuperscript{rd} term is zero.

\[
R_{mp} \frac{R_1}{R_2} - R_{np} = 0 \quad \text{or} \quad \frac{R_{np}}{R_{mp}} = \frac{R_1}{R_2}
\]

\[
R_x = R_3 \frac{R_1}{R_2}
\]
Kelvin Double Bridge: 1 to 0.00001 Ω

Four-Terminal Resistor

Four-terminal resistors have current terminals and potential terminals. The resistance is defined as that between the potential terminals, so that contact voltage drops at the current terminals do not introduce errors.

Four-Terminal Resistor and Kelvin Double Bridge

- $r_1$ causes no effect on the balance condition.
- The effects of $r_2$ and $r_3$ could be minimized, if $R_1 \gg r_2$ and $R_a \gg r_3$.
- The main error comes from $r_4$, even though this value is very small.
Kelvin Double Bridge: 1 to 0.00001 Ω

- 2 ratio arms: $R_1$-$R_2$ and $R_a$-$R_b$
- the connecting lead between $m$ and $n$: yoke

The balance conditions: $V_{lk} = V_{mp}$ or $V_{ok} = V_{onp}$

$$V_{lk} = \frac{R_2}{R_1 + R_2} V \quad (1)$$

here $V = IR_{lo} = I[R_3 + R_x + (R_a + R_b) // R_y]$

$$V_{mp} = I \left[ R_3 + \frac{R_y}{R_a + R_b + R_y} R_b \right] \quad (2)$$

Eq. (1) = (2) and rearrange:

$$R_x = R_3 \frac{R_1}{R_2} + \frac{R_b R_y}{R_a + R_b + R_y} \left( \frac{R_1}{R_2} - \frac{R_a}{R_b} \right)$$

If we set $R_1/R_2 = R_a/R_b$, the second term of the right hand side will be zero, the relation reduce to the well known relation. In summary, The resistance of the yoke has no effect on the measurement, if the two sets of ratio arms have equal resistance ratios.
High Resistance Measurement

Guard ring technique:

- Volume resistance, $R_v$
- Surface leakage resistance, $R_s$

(a) Circuit that measures insulation volume resistance in parallel with surface leakage resistance

$$R_{\text{meas}} = R_s \parallel R_v = \frac{V}{I_s + I_v}$$

(b) Use of guard ring to measure only volume resistance

$$R_{\text{meas}} = R_v = \frac{V}{I_v}$$
High Resistance Measurement

Example  The Insulation of a metal-sheath electrical cable is tested using 10,000 V supply and a microammeter. A current of 5 µA is measured when the components are connected without guard wire. When the circuit is connect with guard wire, the current is 1.5 µA. Calculate (a) the volume resistance of the cable insulation and (b) the surface leakage resistance

SOLUTION

(a ) Volume resistance:

\[ I_V = 1.5 \text{ } \mu A \]

\[ R_V = \frac{V}{I_V} = \frac{10000 \text{ } V}{1.5 \text{ } \mu A} = 6.7 \times 10^9 \text{ } \Omega \]

(b ) Surface leakage resistance:

\[ I_V + I_S = 5 \text{ } \mu A \]

\[ I_S = 5 \text{ } \mu A - I_V = 3.5 \text{ } \mu A \]

\[ R_S = \frac{V}{I_S} = \frac{10000 \text{ } V}{3.5 \text{ } \mu A} = 2.9 \times 10^9 \text{ } \Omega \]
MegaOhm Bridge

Just as low-resistance measurements are affected by series lead impedance, high-resistance measurements are affected by shunt-leakage resistance.

The guard terminal is connected to a bridge corner such that the leakage resistances are placed across bridge arm with low resistances.

\[
\begin{align*}
R_1 \parallel R_C &\approx R_C \quad \text{since } R_1 >> R_C \\
R_2 \parallel R_g &\approx R_g \quad \text{since } R_2 >> R_g \\
\end{align*}
\]

\[
R_x \approx R_A \frac{R_C}{R_B}
\]
**Capacitor**

Capacitance – the ability of a dielectric to store electrical charge per unit voltage

\[ C = \frac{A \varepsilon_0 \varepsilon_r}{d} \]

**Typical values** pF, nF or µF

<table>
<thead>
<tr>
<th>Dielectric</th>
<th>Construction</th>
<th>Capacitance</th>
<th>Breakdown, V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>Meshed plates</td>
<td>10-400 pF</td>
<td>100 (0.02-in air gap)</td>
</tr>
<tr>
<td>Ceramic</td>
<td>Tubular</td>
<td>0.5-1600 pF</td>
<td>500-20,000</td>
</tr>
<tr>
<td></td>
<td>Disk</td>
<td>1pF to 1 µF</td>
<td></td>
</tr>
<tr>
<td>Electrolytic</td>
<td>Aluminum</td>
<td>1-6800 µF</td>
<td>10-450</td>
</tr>
<tr>
<td></td>
<td>Tantalum</td>
<td>0.047 to 330 µF</td>
<td>6-50</td>
</tr>
<tr>
<td>Mica</td>
<td>Stacked sheets</td>
<td>10-5000 pF</td>
<td>500-20,000</td>
</tr>
<tr>
<td>Paper</td>
<td>Rolled foil</td>
<td>0.001-1 µF</td>
<td>200-1,600</td>
</tr>
<tr>
<td>Plastic film</td>
<td>Foil or Metallized</td>
<td>100 pF to 100 µF</td>
<td>50-600</td>
</tr>
</tbody>
</table>
Inductor

Inductance – the ability of a conductor to produce induced voltage when the current varies.

\[ L = \frac{\mu_o \mu_r N^2 A}{l} \]

\( \mu_o = 4\pi \times 10^{-7} \text{ H/m} \)

\( \mu_r \) – relative permeability of core material
- Ni ferrite: \( \mu_r > 200 \)
- Mn ferrite: \( \mu_r > 2,000 \)

Equivalent circuit of an RF coil

Distributed capacitance \( C_d \) between turns

Air core inductor

Iron core inductor
Quality Factor of Inductor and Capacitor

Equivalent circuit of capacitance

Parallel equivalent circuit

Series equivalent circuit

Equivalent circuit of Inductance

Series equivalent circuit

Parallel equivalent circuit

\[ R_p = \frac{R_s^2 + X_s^2}{R_s} \]

\[ X_p = \frac{R_s^2 + X_s^2}{X_s} \]

\[ R_s = \frac{R_p X_p^2}{R_p^2 + X_p^2} \]

\[ X_s = \frac{X_p R_p^2}{R_p^2 + X_p^2} \]
Quality Factor of Inductor and Capacitor

Quality factor of a coil: the ratio of reactance to resistance (frequency dependent and circuit configuration)

Inductance series circuit: \[ Q = \frac{X_s}{R_s} = \frac{\omega L_s}{R_s} \]

Typical \( Q \sim 5 - 1000 \)

Inductance parallel circuit: \[ Q = \frac{R_p}{X_p} = \frac{R_p}{\omega L_p} \]

Dissipation factor of a capacitor: the ratio of reactance to resistance (frequency dependent and circuit configuration)

Capacitance parallel circuit: \[ D = \frac{X_p}{R_p} = \frac{1}{\omega C_p R_p} \]

Typical \( D \sim 10^{-4} - 0.1 \)

Capacitance series circuit: \[ D = \frac{R_s}{X_s} = \omega C_s R_s \]
Inductor and Capacitor

\[ L_S = \frac{R_p^2}{R_p^2 + \omega^2 L_p^2} \cdot L_p \]

\[ R_S = \frac{\omega^2 L_p^2}{R_p^2 + \omega^2 L_p^2} \cdot R_p \]

\[ Q = \frac{\omega L_S}{R_S} \]

\[ C_S = \frac{1 + \omega^2 C_p^2 R_p^2}{\omega^2 C_p^2 R_p^2} \cdot C_p \]

\[ R_S = \frac{1}{1 + \omega^2 C_p^2 R_p^2} \cdot R_p \]

\[ D = \omega C_S R_S \]

\[ I \]

\[ V \]

\[ \frac{V}{R_p} \]

\[ \frac{V}{\omega L_p} \]

\[ I \]

\[ V \]

\[ \frac{V}{R_p} \]

\[ \frac{V}{\omega C_p} \]

\[ D = \frac{1}{\omega C_p R_p} \]
AC Bridge: Balance Condition

- All four arms are considered as impedance (frequency dependent components)
- The detector is an ac responding device: headphone, ac meter
- Source: an ac voltage at desired frequency

$Z_1$, $Z_2$, $Z_3$ and $Z_4$ are the impedance of bridge arms

At balance point:

\[ \frac{E_B}{E_C} = \frac{I_1 Z_1}{I_2 Z_2} \]

\[ I_1 = \frac{V}{Z_1 + Z_3} \quad \text{and} \quad I_2 = \frac{V}{Z_2 + Z_4} \]

General Form of the ac Bridge

Complex Form:

\[ Z_1Z_4 = Z_2Z_3 \]

Polar Form:

\[ Z_1Z_4 (\angle \theta_1 + \angle \theta_4) = Z_2Z_3 (\angle \theta_2 + \angle \theta_3) \]

Magnitude balance:

\[ Z_1Z_4 = Z_2Z_3 \]

Phase balance:

\[ \angle \theta_1 + \angle \theta_4 = \angle \theta_2 + \angle \theta_3 \]
**Example** The impedance of the basic ac bridge are given as follows:

\[ Z_1 = 100 \text{ } \Omega \angle 80^\circ \text{ (inductive impedance)} \]
\[ Z_2 = 250 \text{ } \Omega \text{ (pure resistance)} \]
\[ Z_3 = 400 \angle 30^\circ \Omega \text{ (inductive impedance)} \]
\[ Z_4 = \text{unknown} \]

Determine the constants of the unknown arm.

**SOLUTION** The first condition for bridge balance requires that

\[ Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{250 \times 400}{100} = 1,000 \text{ } \Omega \]

The second condition for bridge balance requires that the sum of the phase angles of opposite arms be equal, therefore

\[ \angle \theta_4 = \angle \theta_2 + \angle \theta_3 - \angle \theta_1 = 0 + 30 - 80 = -50^\circ \]

Hence the unknown impedance \( Z_4 \) can be written in polar form as

\[ Z_4 = 1,000 \text{ } \Omega \angle -50^\circ \]

Indicating that we are dealing with a capacitive element, possibly consisting of a series combination of a resistor and a capacitor.
**Example** an ac bridge is in balance with the following constants: arm AB, \( R = 200 \ \Omega \) in series with \( L = 15.9 \text{ mH} \) \( R \); arm BC, \( R = 300 \ \Omega \) in series with \( C = 0.265 \ \mu \text{F} \); arm CD, unknown; arm DA, \( = 450 \ \Omega \). The oscillator frequency is 1 kHz. Find the constants of arm CD.

\[
\begin{align*}
Z_1 &= R + j\omega L = 200 + j100 \ \Omega \\
Z_2 &= R + \frac{1}{j\omega C} = 300 - j600 \ \Omega \\
Z_3 &= R = 450 \ \Omega \\
Z_4 &= \text{unknown}
\end{align*}
\]

The general equation for bridge balance states that \( Z_1 Z_4 = Z_2 Z_3 \)

\[
Z_4 = \frac{Z_2 Z_3}{Z_1} = \frac{450 \times (200 + j100)}{(300 - j600)} = j150 \ \Omega
\]

This result indicates that \( Z_4 \) is a pure inductance with an inductive reactance of 150 \( \Omega \) at a frequency of 1 kHz. Since the inductive reactance \( X_L = 2\pi fL \), we solve for \( L \) and obtain \( L = 23.9 \text{ mH} \)
Comparison Bridge: Capacitance

Measure an unknown inductance or capacitance by comparing with it with a known inductance or capacitance.

At balance point: \( Z_1Z_x = Z_2Z_3 \)

where \( Z_1 = R_1 \); \( Z_2 = R_2 \); and \( Z_3 = R_3 + \frac{1}{j\omega C_3} \)

\[
R_1 \left( R_x + \frac{1}{j\omega C_x} \right) = R_2 \left( R_3 + \frac{1}{j\omega C_3} \right)
\]

\[
R_x = \frac{R_2 R_3}{R_1} \quad \text{and} \quad C_x = C_3 \frac{R_1}{R_2}
\]

Frequency independent

To satisfy both balance conditions, the bridge must contain two variable elements in its configuration.
Comparison Bridge: Inductance

Measure an unknown inductance or capacitance by comparing with it with a known inductance or capacitance.

At balance point: \( Z_1 Z_x = Z_2 Z_3 \)

where \( Z_1 = R_1; Z_2 = R_2; \) and \( Z_3 = R_3 + j\omega L_3 \)

\[
R_1 \left( R_x + j\omega L_x \right) = R_2 \left( R_S + j\omega L_S \right)
\]

\[
R_x = \frac{R_2 R_3}{R_1} \quad \text{and} \quad L_x = L_3 \frac{R_2}{R_1}
\]

Frequency independent

To satisfy both balance conditions, the bridge must contain two variable elements in its configuration.
Maxwell Bridge

Measure an unknown inductance in terms of a known capacitance

At balance point:

\[ Z_x = Z_2 Z_3 Y_1 \]

where \( Z_2 = R_2 \); \( Z_3 = R_3 \); and \( Y_1 = \frac{1}{R_1} + j\omega C_1 \)

\[ Z_x = R_x + j\omega L_x = R_2 R_3 \left( \frac{1}{R_1} + j\omega C_1 \right) \]

Frequency independent

Suitable for Medium \( Q \) coil (1-10), impractical for high \( Q \) coil: since \( R_1 \) will be very large.
Hay Bridge

Similar to Maxwell bridge: but $R_1$ series with $C_1$

At balance point: $Z_1 Z_x = Z_2 Z_3$

where $Z_1 = R_1 - \frac{j}{\omega C_1}$; $Z_2 = R_2$; and $Z_3 = R_3$

\[
\left( R_1 + \frac{1}{j\omega C_1} \right) (R_x + j\omega L_x) = R_2 R_3
\]

which expands to

\[
R_1 R_x + \frac{L_x}{C_1} - \frac{jR_x}{\omega C_1} + j\omega L_x R_1 = R_2 R_3
\]

Solve the above equations simultaneously
Hay Bridge: continues

\[ R_x = \frac{\omega^2 C_1^2 R_1 R_2 R_3}{1 + \omega^2 C_1^2 R_1^2} \quad \text{and} \quad L_x = \frac{R_2 R_3 C_1}{1 + \omega^2 C_1^2 R_1^2} \]

\[
\tan \theta_L = \frac{X_L}{R} = \frac{\omega L_x}{R_x} = Q
\]
\[
\tan \theta_C = \frac{X_C}{R} = \frac{1}{\omega C_1 R_1}
\]
\[
\tan \theta_L = \tan \theta_C \quad \text{or} \quad Q = \frac{1}{\omega C_1 R_1}
\]

Thus, \( L_x \) can be rewritten as

\[
L_x = \frac{R_2 R_3 C_1}{1 + \left(\frac{1}{Q^2}\right) R_1^2}
\]

For high \( Q \) coil (> 10), the term \((1/Q)^2\) can be neglected

\[
L_x \approx R_2 R_3 C_1
\]
Schering Bridge

Used extensively for the measurement of capacitance and the quality of capacitor in term of $D$

At balance point: 

$$Z_x = Z_2 Z_3 Y_1$$

where $Z_2 = R_2$; $Z_3 = \frac{1}{j\omega C_3}$; and $Y_1 = \frac{1}{R_1} + j\omega C$

$$R_x - \frac{j}{\omega C_x} = R_2 \left( \frac{-j}{\omega C_x} \right) \left( \frac{1}{R_1} + j\omega C_1 \right)$$

which expands to

$$R_x = R_2 \frac{C_1}{C_3} - \frac{jR_2}{\omega C_3 R_1}$$

Separation of the real and imaginary terms yields:

$$R_x = R_2 \frac{C_1}{C_3} \quad \text{and} \quad C_x = C_3 \frac{R_1}{R_2}$$

Diagram of Schering Bridge
Dissipation factor of a series $RC$ circuit:

$$D = \frac{R_x}{X_x} = \omega R_x C_x$$

Dissipation factor tells us about the quality of a capacitor, how close the phase angle of the capacitor is to the ideal value of $90^\circ$.

For Schering Bridge:

$$D = \omega R_x C_x = \omega R_1 C_1$$

For Schering Bridge, $R_1$ is a fixed value, the dial of $C_1$ can be calibrated directly in $D$ at one particular frequency.
Wien Bridge

Measure frequency of the voltage source using series RC in one arm and parallel RC in the adjoining arm

At balance point: \[ Z_2 = Z_1 Z_4 Y_3 \]

\[ Z_1 = R_1 + \frac{1}{j \omega C_1} ; Z_2 = R_2 ; Y_3 = \frac{1}{R_3} + j \omega C_3 ; \text{and} \ Z_4 = R_4 \]

\[ R_2 = \left( R_1 - \frac{j}{\omega C_1} \right) R_4 \left( \frac{1}{R_3} + j \omega C_3 \right) \]

Diagram of Wien Bridge

which expands to

\[ R_2 = \frac{R_1 R_4}{R_3} + j \omega C_3 R_1 R_4 - \frac{j R_4}{\omega C_1 R_3} + \frac{R_4 C_3}{C_1} \]

Rearrange Eq. (2) gives

In most, Wien Bridge, \( R_1 = R_3 \) and \( C_1 = C_3 \)

\[ f = \frac{1}{2 \pi \sqrt{C_1 C_3 R_1 R_3}} \]

\[ (1) \rightarrow R_2 = 2 R_4 \]

\[ (2) \rightarrow f = \frac{1}{2 \pi RC} \]
Wagner Ground Connection

Diagram of Wagner ground

- One way to control stray capacitances is by shielding the arms, reduce the effect of stray capacitances but cannot eliminate them completely.
  - Wagner ground
  - Stray across arm
  - Cannot eliminate

- Wagner ground connection eliminates some effects of stray capacitances in a bridge circuit.
  - Simultaneous balance of both bridge makes the point 1 and 2 at the ground potential. (short $C_1$ and $C_2$ to ground, $C_4$ and $C_5$ are eliminated from detector circuit)
  - The capacitance across the bridge arms e.g. $C_6$ cannot be eliminated by Wagner ground.
Ceramic Capacitor

### Capacitor Values

<table>
<thead>
<tr>
<th>Multiplier</th>
<th>Tolerance of Capacitor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>For the Number</strong></td>
<td><strong>Multiplier</strong></td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>1,000</td>
</tr>
<tr>
<td>4</td>
<td>10,000</td>
</tr>
<tr>
<td>5</td>
<td>100,000</td>
</tr>
<tr>
<td>8</td>
<td>0.01</td>
</tr>
<tr>
<td>9</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Examples:**

- 152K = 15 × 100 = 1500 pF or 0.0015 μF, ±10%
- 759J = 75 × 0.1 = 7.5 pF, ±5%

**Note:** The letter R may be used at times to signify a decimal point, as in 2R2 = 2.2 (pF or μF).
Capacitor Values

Film Capacitor
Capacitor Values

Chip Capacitor

Alternate Two-Place Code
* Values below 100 pF—Value read directly

05 = 5 pF
82 = 82 pF

* Values 100 pF and above—Letter/number code

A1 = 10 \times 10 = 100 pF
N3 = 33 \times 1000 = 33000 pF

Multiplier (1–9)
Value (1st and 2nd significant digits)

Value (24 Value Symbols)—Uppercase Letters Only

A-10
B-11
C-12
D-13
E-15
F-16
G-18
H-20
J-22
K-24
L-27
M-30
N-33
P-36
Q-39
R-43
S-47
T-51
U-56
V-62
W-68
X-75
Y-82
Z-91

Multiplier
1 = \times 10
2 = \times 100
3 = \times 1000
4 = \times 10,000
5 = \times 100,000 etc.

Chip capacitor coding system.

Standard Single-Place Code
Orange

= 4.7 \times 1.0 = 4.7 pF

Color multiplier
Symbol value

Examples: R (Green) = 3.3 \times 100 = 330 pF
7 (Blue) = 8.2 \times 1000 = 8200 pF

Value (24 Value Symbols)—Uppercase Letters and Numerals

A-1.0
B-1.1
C-1.2
D-1.3
E-1.5
H-1.6
I-1.8
J-2.0
K-2.2
L-2.4
N-2.7
O-3.0
P-3.3
Q-3.6
R-3.9
S-3.6
T-3.9
V-4.3
W-4.7
X-5.1
Y-5.6
Z-6.2

Multiplier
Orange = \times 1.0
Black = \times 10
Green = \times 100
Blue = \times 1000
Violet = \times 10,000
Red = \times 100,000
Capacitor Values

Tantalum Capacitor

<table>
<thead>
<tr>
<th>Color</th>
<th>Rated Voltage</th>
<th>Capacitance in Picofarads</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st Figure</td>
<td>2nd Figure</td>
</tr>
<tr>
<td>Black</td>
<td>4</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Brown</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Red</td>
<td>10</td>
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<tr>
<td>Orange</td>
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<td>Yellow</td>
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<td>Gray</td>
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</tr>
<tr>
<td>White</td>
<td>3</td>
<td>9</td>
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</tbody>
</table>
Capacitor Values

Chip Capacitor

<table>
<thead>
<tr>
<th>Value (33 Value Symbols)—Upper and Lowercase Letters</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-1.0</td>
<td>X-7.5</td>
</tr>
<tr>
<td>B-1.1</td>
<td>T-5.1</td>
</tr>
<tr>
<td>C-1.2</td>
<td>U-5.6</td>
</tr>
<tr>
<td>D-1.3</td>
<td>m-6.0</td>
</tr>
<tr>
<td>E-1.5</td>
<td>y-9.0</td>
</tr>
<tr>
<td>F-1.6</td>
<td>Z-9.1</td>
</tr>
<tr>
<td>G-1.8</td>
<td>etc.</td>
</tr>
</tbody>
</table>

- 0 = \times 1.0
- 1 = \times 10
- 2 = \times 100
- 3 = \times 1,000
- 4 = \times 10,000
- 5 = \times 100,000

Multiplier (0-9)

Value (1st and 2nd capacitance digits)

2.2 \times 1000
2200 \text{ pF}