Velocity Transducer

Use the principle of electromagnetic induction: linear and angular velocity transducer

![Diagram: Linear and angular velocity measurement]

Basic equation relating voltage generated to velocity of a conductor in a magnetic field can be expressed as

$$V_T = Blv$$

- $V_T$ = the voltage generated by the transducer
- $B$ = the component of the flux density normal to the velocity
- $l$ = the length of the conductor
- $v$ = the velocity
Velocity Transducer

LVT is equivalent to a voltage generated connected in series with an inductance $L_T$ and a resistance $R_T$ and here $R_M$ is the input resistance of a recording instrument,

$$L_T \frac{di}{dt} + (R_T + R_M) i = V_T = S_v v$$

$S_v = \text{the voltage sensitivity (mV/(in/s))}$
$v = \text{the time dependent velocity (in/s)}$
$i = \text{the current flowing in the circuit}$

Assume a sinusoidal input velocity, the frequency response can be obtained

$$V_o(i\omega) = i(i\omega)R_M = \frac{R_M S_v v \angle \phi}{\sqrt{(R_T + R_M)^2 + (\omega L)^2}}$$

here $\phi = \arctan\left(-\frac{\omega L}{R_T + R_M}\right)$
Most accelerometers use the mass-spring-damper system, under a steady acceleration, the mass will move, stretching or compressing the spring until the force exerted by spring balance the force by the force due to acceleration

\[ k_m \Delta y = ma \]

\[ \Delta y = \frac{m}{k_m} a \]

Here \( \frac{m}{k_m} = \frac{1}{\omega_n^2} \) = static sensitivity

So the measurement of steady acceleration is just a displacement problem.

For dynamic behavior: the system is a second order system

\[ my'' + b_m (y' - x_{in}') + k_m (y - x_{in}) = 0 \]

Let \( z = y - x_{in} \)

\[ mz'' + b_m z' + k_m z = -mx_{in}'' = -F \]

\[ \omega_n = \sqrt{\frac{k_m}{m}} \]

= natural frequency

\[ \zeta = \frac{b_m}{2 \sqrt{k_m m}} \]

= damping ratio
Accelerometer using a potentiometer

\[ q = S_q F \]

\[ V = \frac{q}{C} = \frac{S_q F}{C} \]

\[ m z'' + b_m z' + k_m z = -m x_{in}'' = -F \]

\[ D = \text{piezoelectric strain constant} \]
Piezoelectric Effect

A piezoelectric material produces an electric charge when its subject to a force or pressure. The piezoelectric materials such as quartz or polycrystalline barium titanate, contain molecules with asymmetrical charge distribution. Therefore, under pressure, the crystal deforms and there is a relative displacement of the positive and negative charges within the crystal.

Cubic unit cell has a center of symmetry

Hexagonal unit cell has no center of symmetry

Piezoelectric Effect

Charge, \( q \) develops can be determined from the output \( V_o \)

\[
q = V_o C = S_q F = S_q A P
\]

- \( C \) = capacitance
- \( S_q \) = charge sensitivity
- \( A \) = area
- \( P \) = applied pressure
- \( d \) = distance between electrode

Quartz: Young’s modulus 86 GPa, resistivity \( 10^{12} \) \( \Omega \).m and dielectric constant = 40.6 pF/m

| Table 5.3 Typical Charge and Voltage Sensitivities, \( S_q \) and \( S_v \), of Piezoelectric Materials |
|---------------------------------|---------------------------------|------------------|------------------|
| Material                        | Orientation                     | \( S_q (pC/N) \)  | \( S_v (V \cdot m/N) \) |
| Quartz SiO₂                     | X-cut, length longitudinal      | 2.2              | 0.055            |
| Single crystal                  | X-cut, thickness longitudinal   | -2.0             | -0.05            |
|                                 | Y-cut, thickness shear          | 4.4              | 0.11             |
| Barium titanate                 | Parallel to polarization        | 130              | 0.011            |
| BaTiO₃                          | Perpendicular to polarization   | -56              | -0.004           |
Piezoelectric Effect

Schematic diagram of a measuring system with a piezoelectric sensor
Pressure Transducer

Pressure transducers - use some form of mechanical device that stretches proportionally in response to an applied pressure. Strain gages, LVDT, potentiometers, variable inductance, or capacitance convert this displacement into an electrical signal.
Bourdon tube is a curve metal tube having an elliptical cross section that mechanically deforms under pressure.
Bellow is a thin-walled, flexible metal tube formed into deep convolutions and seal at one end.
Diaphragm is a thin elastic circular plate supported about its circumference.
Pressure Transducer

Capacitive pressure sensor

Diaphragm pressure sensor

Capacitive pressure sensor
Flow Transducer

Volume flow rate: \[ Q = \frac{dV}{dt} \]

Mass flow rate: \[ Q_m = \frac{dm}{dt} = \rho Q \]

Velocity: \[ v = \frac{Q}{A} \]

Where \( \rho \) is the density of fluid and \( A \) is the cross section of the pipe

Restriction Flow sensors

An intentional reduction in flow will cause a measurable pressure drop across the flow path

\[ Q = k_2 \sqrt{P_2 - P_1} \]

\( Q \) = Volumetric flow rate
\( k \) = Constant is set by the geometry
\( P_2 \) = high-side pressure
\( P_1 \) = low-side pressure
Flow Transducer

Deflection type Flow sensor

Spin type Flow sensor

Electromagnetic Flow sensor
Level Transducer

Continuous level:
indicate the precise level, proportionally along the entire height of the tank

Discrete level
indicate only when the tank reaches the predefined level

Discrete level transducer
Level Transducer

Level measurement by pressure sensor

Level measurement by differential pressure sensor

Level measurement by force sensor

Capacitive level sensor
Temperature Transducer

<table>
<thead>
<tr>
<th>Advantages</th>
<th>RTD Advantages</th>
<th>Thermistor Advantages</th>
<th>Integrated circuit sensor Advantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Self powered</td>
<td>- Most stable</td>
<td>- High output</td>
<td>- Most linear</td>
</tr>
<tr>
<td>- Simple</td>
<td>- Most accurate</td>
<td>- fast</td>
<td>- Highest output</td>
</tr>
<tr>
<td>- Rugged</td>
<td>- More linear than thermocouple</td>
<td>- two-wire ohms measurement</td>
<td>- Inexpensive</td>
</tr>
<tr>
<td>- Inexpensive</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Wide variety</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- Wide temperature range</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disadvantages</th>
<th>RTD Disadvantages</th>
<th>Thermistor Disadvantages</th>
<th>Integrated circuit sensor Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>- Nonlinear</td>
<td>- Expensive</td>
<td>- Nonlinear</td>
<td>- $T&lt;200^\circ{}C$</td>
</tr>
<tr>
<td>- Low voltage</td>
<td>- Current source required</td>
<td>- Limited temperature range</td>
<td>- Power supply required</td>
</tr>
<tr>
<td>- Reference required</td>
<td>- Small $\Delta R$</td>
<td>- Fragile</td>
<td>- Slow</td>
</tr>
<tr>
<td>- Least stable</td>
<td>- Low absolute resistance</td>
<td>- Current source required</td>
<td>- Self-heating</td>
</tr>
<tr>
<td>- Least sensitive</td>
<td>- Self-heating</td>
<td></td>
<td>- Limited configurations</td>
</tr>
</tbody>
</table>

**Figure 11.49** Advantages and disadvantages of the most common temperature sensors. (Courtesy of Omega Engineering, Inc.)
Thermocouple:
a simple temperature sensor consists of two dissimilar materials in thermal contact (junction), the electrical potential (Seebeck voltage) is developed that is proportional to the temperature of the junction.

\[ V = s \Delta T \]

\( s \): Thermoelectric coefficient (material dependence)
# Thermocouple

## TABLE 7.2  Standard Thermocouple Types and Useful Temperature Range

<table>
<thead>
<tr>
<th>Letter Designation</th>
<th>Metals</th>
<th>Approximate Temperature Range (degrees Celsius)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type K</td>
<td>Chromel/Alumel</td>
<td>− 200 to 1250</td>
</tr>
<tr>
<td>Type J</td>
<td>Iron/Constantan</td>
<td>0 to 750</td>
</tr>
<tr>
<td>Type T</td>
<td>Copper/Constantan</td>
<td>− 200 to 350</td>
</tr>
<tr>
<td>Type E</td>
<td>Chromel/Constantan</td>
<td>− 200 to 900</td>
</tr>
<tr>
<td>Type S</td>
<td>Platinum/Platinum 10% Rhodium</td>
<td>0 to 1450</td>
</tr>
<tr>
<td>Type R</td>
<td>Platinum/Platinum 13% Rhodium</td>
<td>0 to 1450</td>
</tr>
</tbody>
</table>
Thermocouple

In practice, we can't measure Seebeck voltage directly because we must connect voltmeter to the thermometer, and the voltmeter leads themselves create a new thermoelectric circuit.

How can we know the temperature at J₁?

\[ V_3 = 0 \]

Equivalent circuit

\[ V = V_1 - V_2 \]

Equivalent circuit
The thermoelectric circuit is used to sensed an unknown Temperature $T_1$, while junction 2 is maintained at a known reference temperature $T_2$. It is possible to determine $T_1$ by measuring voltage $V$.

Accurate conversion of the output voltage $V$, to $T_1 - T_2$ is achieved either by using \textit{calibration (lookup) tables} or by using \textit{a higher order polynomial}.

$$T_1 - T_2 = a_0 + a_1 V + a_2 V^2 + \cdots + a_n V^n$$

Where $a_0$, $a_1$, $\ldots$, $a_n$ are coefficients specified for each pair of thermocouple materials, and $T_1 - T_2$ is the difference temperature in °C.
Principles of Thermocouple Behavior

1. A thermocouple circuit must contain at least two dissimilar materials and at least two junctions.
2. The output voltage $V_o$ of a thermocouple circuit depends only on the difference between junction temperatures ($T_1 - T_2$) and is independent of the temperatures elsewhere in the circuit if no current flows in the circuit.
3. If a third metal C is inserted into either leg (A or B) of a thermocouple circuit, the output voltage $V_o$ is not affected, provided that two new junctions (A/C and C/A) are maintained at the same temperature, for example, $T_i = T_j = T_3$.
Principles of Thermocouple Behavior

• The insertion of an intermediate metal C into junction 1 does not affect the output voltage $V_o$, provided that the two junctions formed by insertion of the intermediate (A/C and C/B) are maintained at the same temperature $T_1$.

• A Thermocouple circuit with temperatures $T_1$ and $T_2$ produces an output voltage $(V_o)_{1-2} = f(T_1 - T_2)$, and one exposed to temperatures $T_2$ and $T_3$ produces an output voltage $(V_o)_{2-3} = f(T_2 - T_3)$. If the same circuit is exposed to temperatures $T_1$ and $T_3$, the output voltage $(V_o)_{1-3} = f(T_1 - T_3) = (V_o)_{1-2} + (V_o)_{2-3}$.

(d) Intermediate metal in junction

(e) Voltage addition from identical thermocouples at different temperatures
Principles of Thermocouple Behavior

- A thermocouple circuit fabricated from materials A and C generates an output voltage \((V_o)_{A/C}\) when exposed to temperatures \(T_1\) and \(T_2\), and a similar circuit fabricated from materials C and B generates an output voltage \((V_o)_{C/B}\). Furthermore, a thermocouple fabricated from materials A and B generates an output voltage \((V_o)_{A/B} = (V_o)_{A/C} + (V_o)_{C/B}\)

(f) Voltage addition from different thermocouples at identical temperatures
# Thermocouple

## Table A.6 Polynomial Coefficients for Six Different Types of Thermocouples

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Type of Thermocouple</th>
<th>Temperature Range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>−100° to 1000°C</td>
</tr>
<tr>
<td></td>
<td>J</td>
<td>0° to 760°C</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>0° to 1370°C</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>0° to 1000°C</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>0° to 1750°C</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>−160° to 400°C</td>
</tr>
<tr>
<td>$a_0$</td>
<td>0.104967248</td>
<td>0.048868252</td>
</tr>
<tr>
<td>$a_1$</td>
<td>17189.45282</td>
<td>19873.14503</td>
</tr>
<tr>
<td>$a_2$</td>
<td>−282639.0850</td>
<td>−218614.5353</td>
</tr>
<tr>
<td>$a_3$</td>
<td>12695339.5</td>
<td>11569199.78</td>
</tr>
<tr>
<td>$a_4$</td>
<td>−448703084.6</td>
<td>−264917531.4</td>
</tr>
<tr>
<td>$a_5$</td>
<td>1.10866E +10</td>
<td>2018441314</td>
</tr>
<tr>
<td>$a_6$</td>
<td>−1.76807E +11</td>
<td>−1.18452E +12</td>
</tr>
<tr>
<td>$a_7$</td>
<td>1.71842E +12</td>
<td>1.38690E +13</td>
</tr>
<tr>
<td>$a_8$</td>
<td>−9.19278E +12</td>
<td>−6.33708E +13</td>
</tr>
<tr>
<td>$a_9$</td>
<td>2.06132E +13</td>
<td>1.69535E +20</td>
</tr>
</tbody>
</table>
# Thermocouple

Themoelectric voltages:
- Chromel-Alumel Type K (Table A.2)
- Copper-Constantan Type T (Table A.3)
- Iron-Constantan Type J (Table A.4)

## Table A.2 Thermoelectric Voltages for Chromel-Alumel Thermocouples with the Reference Junction at 0°C (32°F)

<table>
<thead>
<tr>
<th>°C</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>-270</td>
<td>-6.458</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-270</td>
</tr>
<tr>
<td>-200</td>
<td>-5.891</td>
<td>-5.907</td>
<td>-5.922</td>
<td>-5.936</td>
<td>-5.951</td>
<td>-5.965</td>
<td>-5.980</td>
<td>-5.994</td>
<td>-6.007</td>
<td>-6.021</td>
<td>-6.035</td>
<td>-200</td>
</tr>
<tr>
<td>-190</td>
<td>-5.730</td>
<td>-5.747</td>
<td>-5.763</td>
<td>-5.780</td>
<td>-5.796</td>
<td>-5.813</td>
<td>-5.829</td>
<td>-5.845</td>
<td>-5.860</td>
<td>-5.876</td>
<td>-5.891</td>
<td>-190</td>
</tr>
<tr>
<td>-170</td>
<td>-5.354</td>
<td>-5.374</td>
<td>-5.394</td>
<td>-5.414</td>
<td>-5.434</td>
<td>-5.454</td>
<td>-5.474</td>
<td>-5.493</td>
<td>-5.512</td>
<td>-5.531</td>
<td>-5.550</td>
<td>-170</td>
</tr>
<tr>
<td>-160</td>
<td>-5.141</td>
<td>-5.163</td>
<td>-5.185</td>
<td>-5.207</td>
<td>-5.228</td>
<td>-5.249</td>
<td>-5.271</td>
<td>-5.292</td>
<td>-5.313</td>
<td>-5.333</td>
<td>-5.354</td>
<td>-160</td>
</tr>
<tr>
<td>-150</td>
<td>-4.912</td>
<td>-4.936</td>
<td>-4.969</td>
<td>-4.993</td>
<td>-5.029</td>
<td>-5.051</td>
<td>-5.074</td>
<td>-5.097</td>
<td>-5.119</td>
<td>-5.141</td>
<td>-5.154</td>
<td>-150</td>
</tr>
</tbody>
</table>
• Using hardware compensation (electronic ice point reference)

• Commercial ICs are available for a wide variety of TC

AD594: Type J (iron-constantan)
AD595: Type K (chromal-alumel)
These ICs give approximate output

\[ V \approx 10 \frac{\text{mV}}{\degree C} T_1 \]
Thermocouple

• Using software compensation

\[ V = V_1 + V_2 + V_3 \]

\[ V = V_{\text{Cu/Constantan}}(T_1) + V_{\text{Cu/Cu}}(T_2) + V_{\text{Constantan/Cu}}(T_2) \]

**Ex** assume that the arbitrary reference temperature \( T_2 \) is maintained at 100°C and that an output voltage \( V = 8.388 \text{ mV} \) is recorded. Find \( T_1 \)

From calibration tables: \( V_{\text{Cu/constantan}}(100^\circ\text{C}) = -V_{\text{constantan/Cu}}(100^\circ\text{C}) = -4.277 \text{ mV} \)

\[ V_{\text{Cu/Constantan}}(T_1) = 8.388 - (-4.277) = 12.665 \text{ mV} \]

From calibration tables: \( V_{\text{Cu/constantan}} = 12.665 \text{ mV} \) would be produced by a temperature of \( T_1 = 261.7^\circ\text{C} \)

• This method relies on a computer program that contained calibration tables of TC

• Thermistor or RTD is used to gain the absolute temp. of reference junction (ambient temperature).
Thermocouple

- Using hardware compensation (electronic ice point reference)

- Commercial ICs for various TC
  - AD594: Type J (iron-constantan)
  - AD595: Type K (chromal-alumel)

These ICs give approximate output

\[ V \approx 10 \frac{\text{mV}}{^\circ\text{C}} T_1 \]
Resistive Temperature Detectors (RTDs)

An RTD: All metals produce positive change in resistance for a positive change in temperature

\[ R = R_0 \left[ 1 + \alpha_1 (T - T_0) + \alpha_2 (T - T_0)^2 + \ldots + \alpha_n (T - T_0)^n \right] \]

Where \( R_0 \) is the resistance at the reference temperature \( T_0 \). \( \alpha_n \) is the temperature coefficient.

ex. For a Pt wire, \( \alpha_1 \approx 3.95 \times 10^{-3}/K, \alpha_2 \approx 5.83 \times 10^{-7}/K^2 \)

For a limited range of temperature, the linear form can be used

\[ R = R_0 \left[ 1 + \alpha_1 (T - T_0) \right] \]

The sensitivity to temperature

\[ S = R_0 \alpha_1 \]

For a Pt wire, this corresponds to a change of only \( \approx 0.4\%/^\circ C \)

Resistance-temperature curves for nickel, copper and platinum.
RTD: Common Errors

- **Lead-wire effects**
  - Use short lead wire \((R_L < 1\% \text{ of } RTD)\)
  - Use three or four lead-wire system

- **Stability**
  - Stability may become a source of error when the upper temperature is exceeded.

- **Self-heating**
  - Self-heating occurs because of the power dissipation in sensor, \(P_D = I^2R_T\)
  - The increase in temperature from self-heating \(\Delta T\) due to \(P_D = I^2R_T\) is:
    \[
P_D = \delta \Delta T
    \]
  - Where \(\delta\) is heat dissipation factor (mW/K)
  - To minimize self-heating effect, the power dissipation must be limited.

- **Sensitivity of the RTD to strain**
  - Normally, this error can be negligible since the strain sensitivity of the sensor is small comparison with the temperature sensitivity.
RTD

\[ V_{DVM} = iR_T \]
Thermistor

Thermistors: temperature-dependent resistors that are based on semiconductor materials such as oxides of nickel, cobalt, or manganese and sulfides or irons, aluminum or copper. They are designated as NTC when having a negative temperature coefficient and as PTC when having a positive temperature coefficient.

**Mechanism:**
Variation of the number of charge carrier and mobility with temperatures

**NTC thermistor:** the dependence of $R$ with temperature is almost exponential:

$$R = R_0 e^{\beta(1/T - 1/T_0)}$$

Where $R_0 = \text{the resistance at the reference temperature } T_0$ and
$\beta = \text{the characteristic temperature, usually ranges from 2000 to 4000 K.}$
$\beta$ is also temperature dependent parameter.
$T$ and $T_0 = \text{absolute temperature, K}$
Thermistor

The equivalent TCR or relative sensitivity:

\[ \alpha = \frac{1}{R} \frac{dR}{dT} = \frac{1}{R} S = -\frac{\beta}{T^2} \]

Which shows a nonlinear dependence on \( T \). At 25°C and taking \( \beta = 4000 \) K, \( \alpha = -4.5\% /K \), which is more than ten times higher than that of PT100 probe (\( \alpha = +0.35\% /K \)). If \( R_o = 2000 \) \( \Omega \) then \( \Delta R/\Delta T = 90 \) \( \Omega/K \). Therefore, the effect of lead resistance is less than in thermistor compared to RTD.

Steinhart-Hart relation:

\[ \frac{1}{T} = A + B \ln R_T + C(\ln R_T)^3 \]

Where \( A, B \) and \( C \) = coefficient determined from calibration curves

Simpler relation:

\[ T = \frac{B}{\ln R - A} - C \]
Integrated-Circuit Temperature Transducer

IC temperature sensors: combine the temperature sensing element and the signal-conditioning electronics

**LM335 outputs:** 10 mV/K or $2.73 \, \text{V} + 10 \times (\text{mV/}^\circ\text{C}) \, T$

**LM34 outputs:** 10 mV/$^\circ$F

**AD592 outputs:** 1 µA/K or $273 \times (\mu\text{A/}^\circ\text{C}) \, T$

\[ V_{out} = \frac{10 \times (\text{mV/}^\circ\text{C})}{R_{bias}} \times T + 2.73 \, \text{V} \]