LECTURE 1
REVIEW OF MICROECONOMIC THEORY I

1. Preferences and Choices

- When **preferences** are complete, reflexive, transitive, and continuous and preferences ordering is monotonic and convex, such preferences can be represented by a **utility function**.
- **The Marginal Rate of Substitution (MRS)** measures the slope of the indifference curve, i.e., it tells us the rate the consumer is willing to substitute between the two goods. Given a utility function $U(x_1, x_2)$,

$$MRS_{x_1,x_2} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2}$$

- A consumer's **optimal choice**, (x_1^*, x_2^*), is at the tangent point between the indifference curve and the budget line. At $\left(x_1^*, x_2^*\right)$,

$$MRS_{x_1,x_2} = -\frac{p_1}{p_2}$$

Note that (x_1^*, x_2^*) can be interior or boundary optimum.

2. Utility Maximisation Problem

$$\max_{x_1,x_2} U\left(x_1, x_2\right)$$

subject to

$$p_1x_1 + p_2x_2 = m$$

We can write the following Lagrangian

$$\mathcal{L} = U\left(x_1, x_2\right) + \lambda\left(m - p_1x_1 - p_2x_2\right)$$

The first-order conditions (FOCs) are

$$\frac{\partial U\left(x_1, x_2\right)}{\partial x_1}=\lambda p_1 \text{ and } \frac{\partial U\left(x_1, x_2\right)}{\partial x_2}=\lambda p_2$$

Or,

$$\begin{aligned}
\frac{\partial U\left(x_1, x_2\right) / \partial x_1}{\partial U\left(x_1, x_2\right) / \partial x_2} &= \frac{p_1}{p_2} \text{ or} \\
MRS_{x_1, x_2} &= \text{price ratio}
\end{aligned}$$

Denote the solutions by $x_1^*(p, m)$ and $x_2^*(p, m)$. We refer to these two equations as the **Marshallian demand function** for good 1 and 2 respectively.

Remark 1 *We implicitly assume that the second-order condition is satisfied but what is the second-order condition for this utility maximisation problem?*

3. Comparative Statics

- When a price changes (say p_1), we can decompose the total effect (or **price effect**) on the demand into (1) income effect and (2) substitution effect.
- **Income effect** is a change in demand due to a change in purchasing power.
- Things get a little complicate for substitution effect. There are two criteria: Slutsky substitution effect and Hicks substitution effect. Note that substitution effect is negative in the sense that it works in the opposite direction of the change in price.
- **Slutsky substitution effect** is a change in demand due to a change in the rate of exchange between the two goods, given that the consumer has the same purchasing power.
- **Hicks substitution effect** is a change in demand due to a

change in the rate of exchange between the two goods, given the same level of utility.

- When a price changes
  - For Slutsky substitution effect, the consumer is compensated just enough to *afford* the original consumption bundle.
  - While for Hicks substitution effect, the consumer is compensated just enough that he/she feels *indifferent* with the original bubdle.
- These two substitution effects have the corresponding demand functions. Hicksian demand function is often refered to as **compensated demand**.
- **The Law of Demand** (from Slutsky equation): "If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases."

- **Derivation of the Slutsky Equation**: given the original bundle (x_1, x_2), we can write the Slutsky demand function (for good 1) at prices (p_1, p_2) as $x_1^s(p, x)$. At other prices (p_1', p_2') and income $m' = p_1'x_1 + p_2'x_2$, the ordinary demand (for good 1) is $x_1(p', m')$. If we adjust this ordinary demand to the previous purchasing power, by the definition of Slutsky substitution effect, we can write

$$x_1^s(p, x) = x_1(p, m)$$

Differentiate this equation with respect to p_1, we have

$$\begin{aligned}
\frac{\partial x_1^s(p, x)}{\partial p_1} &= \frac{\partial x_1(p, m)}{\partial p_1} + \frac{\partial x_1(p, m)}{\partial m} x_1 \text{ or} \\
\frac{\partial x_1(p, m)}{\partial p_1} &= \frac{\partial x_1^s(p, x)}{\partial p_1} - \frac{\partial x_1(p, m)}{\partial m} x_1 \\
\text{Price Effect} &= \text{Substitution Effect} + \text{Income Effect}
\end{aligned}$$

- Derivation of the Slutsky Equation (with Hicks substitution effect): at p' and m' we can write the ordinary demand $x_1(p', m')$. When the price changes from p' to p (and the income changes from m' to m), by definition the new ordinary demand $x_1(p, m)$ is the same as the Hicksian demand at price p and a constant utility level $\overline{u}$,

$$x_1^h(p, \overline{u}) = x_1(p, m)$$

Differentiate this equation with respect to p_1 :

$$\frac{\partial x_1^h(p, \overline{u})}{\partial p_1} = \frac{\partial x_1(p, m)}{\partial p_1} + \frac{\partial x_1(p, m)}{\partial m} x_1 \text{ or}$$

$$\frac{\partial x_1(p, m)}{\partial p_1} = \frac{\partial x_1^h(p, \overline{u})}{\partial p_1} - \frac{\partial x_1(p, m)}{\partial m} x$$

$$\text{Price Effect} = \text{Substitution Effect} + \text{Income Effect}$$

- Holding the prices fixed, at a different level of income we have a different consumption bundle. We can connect these bundles to form the **income offer curve** or **income expansion path**.

- Holding the prices fixed, we can write the ordinary demand as a function of income, says for good 1, $x_1(\bar{p}, m)$. This curve is refered to as the **Engle curve**.

- Holding the price of good 2 and income fixed, we can find optimal bundles for various level of p_1. We can connect these points to form the **price offer curve** of good 1. We can also use the relationship between x_1 and p_1 while holding p_2 and m constant to plot the **demand curve** of good 1.

4. Endowment

- Now we assume that a consumer starts of with an **endowment** of the two goods before enters the market. Denote this endowment by (ω_1, ω_2).

- The **gross demand** (says for good 1) is the amount of good 1 the consumer ends up consuming while the **net demand** (for good 1) is the difference between the gross demand and the initial endowment:

$$\begin{aligned} \text{Gross demand (for good 1)} &: \quad x_1 \geq 0 \\ \text{Net demand (for good 1)} &: \quad x_1 - \omega_1 \gtreqless 0 \end{aligned}$$

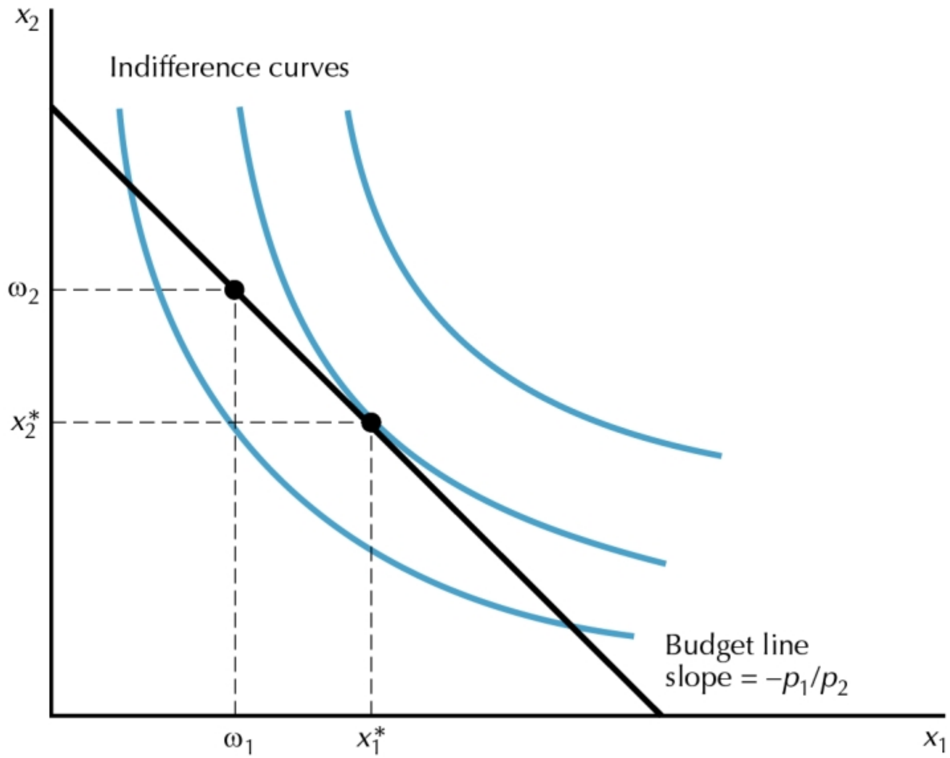
- We can write the budget constraint, taking into account the initial endowment, as follow:

$$m = p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2$$

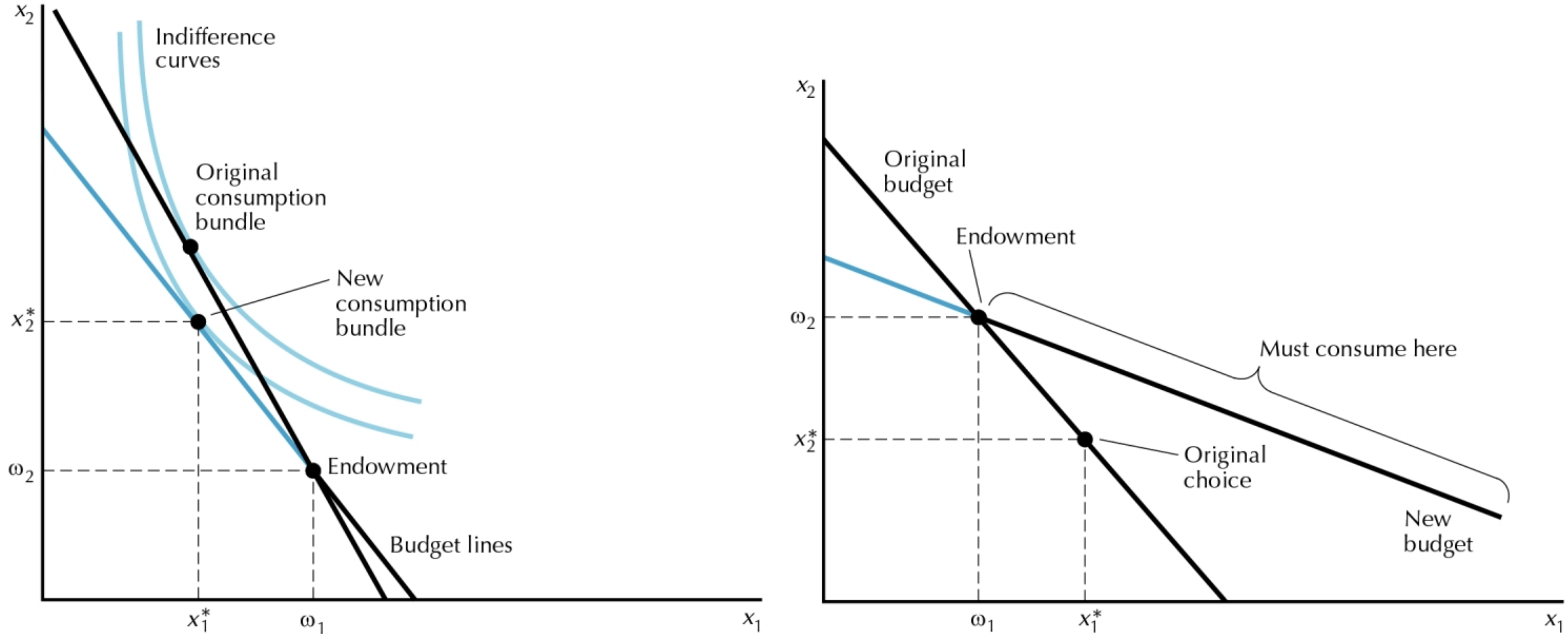
Or in term of net demands for both goods,

$$p_1 (x_1 - \omega_1) + p_2 (x_2 - \omega_2) = 0$$

Remark 2 *Why the initial endowment ends up on the budget line?*



From the figure (9.1 in Varian), this consumer is a supplier of good 2 $(\omega_2 > x_2^*)$ and a buyer of good 1 $(x_1^* > \omega_1)$.



A decrease in the price of good 1 makes the budget line pivot around the initial endowment.
(Figure 9.3-9.4 from Varian)

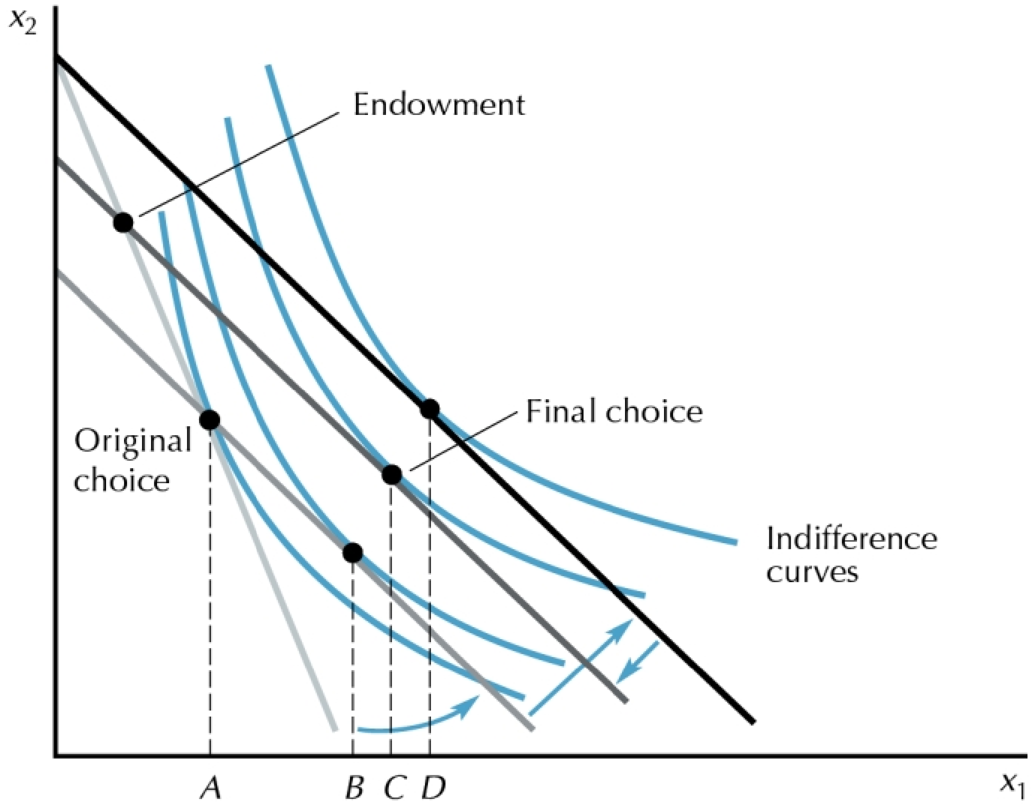
5. Comparative Statics with Endowment

What happens if p_1 changes?

Following the same footsteps we have done, we can decompose the total effect (price effect) into substitution and income effects but with the initial endowment this income effect can be divided into two effects:

- **Ordinary Income Effect** which is the same as we have discussed before;
- **Endowment Income Effect** arises because when the price changes, it affects the value of the initial endowment and hence income.

Therefore, we need to revise the Slutsky equation to take into account this endowment income effect.



From A to C - price effect; from A to B - (Slutsky) substitution effect; from B to D - ordinary income effect; and from D to C - endowment income effect. (Figure 9.7 from Varian)

Derivation of Slutsky Equation (again!)

From the Marshallian demand $x_1(p,m)$, holding p_2 fixed, we take the derivative with respect to p_1

$$\frac{dx_1(p,m)}{dp_1} = \frac{\partial x_1(p,m)}{\partial p_1} + \frac{\partial x_1(p,m)}{\partial m}\frac{\partial m}{\partial p_1}$$

From $m = p_1x_1 + p_2x_2 = p_1\omega_1 + p_2\omega_2$, we have $\frac{\partial m}{\partial p_1} = \omega_1$.

Recall the Slutsky equation we have derived

$$\frac{\partial x_1(p,m)}{\partial p_1} = \frac{\partial x_1^s(p,x)}{\partial p_1} - \frac{\partial x_1(p,m)}{\partial m}x_1$$

Then,

$$
\begin{aligned}
\frac{dx_1(p,m)}{dp_1} &= \frac{\partial x_1^s(p,x)}{\partial p_1} - \frac{\partial x_1(p,m)}{\partial m}x_1 + \frac{\partial x_1(p,m)}{\partial m}\omega_1 \\
&= \frac{\partial x_1^s(p,x)}{\partial p_1} + \frac{\partial x_1(p,m)}{\partial m}(\omega_1 - x_1) \\
\text{Price Effect} &= \text{Substitution Effect} + \text{Endowment Income Effect} \\
& \quad + \text{Ordinary Income Effect}
\end{aligned}
$$

6. Application: Allocation of Time

A worker is endowed with $\overline{R}$ hours that can be used for labour (with a wage rate w per hour) or leisure and an income $m, \omega = (\overline{R}, m)$. This worker also purchases a consumption good C at a price p_C.

We can write the budget constraint of this worker as

$$p_C C = w\left(\overline{R} - R\right) + m$$

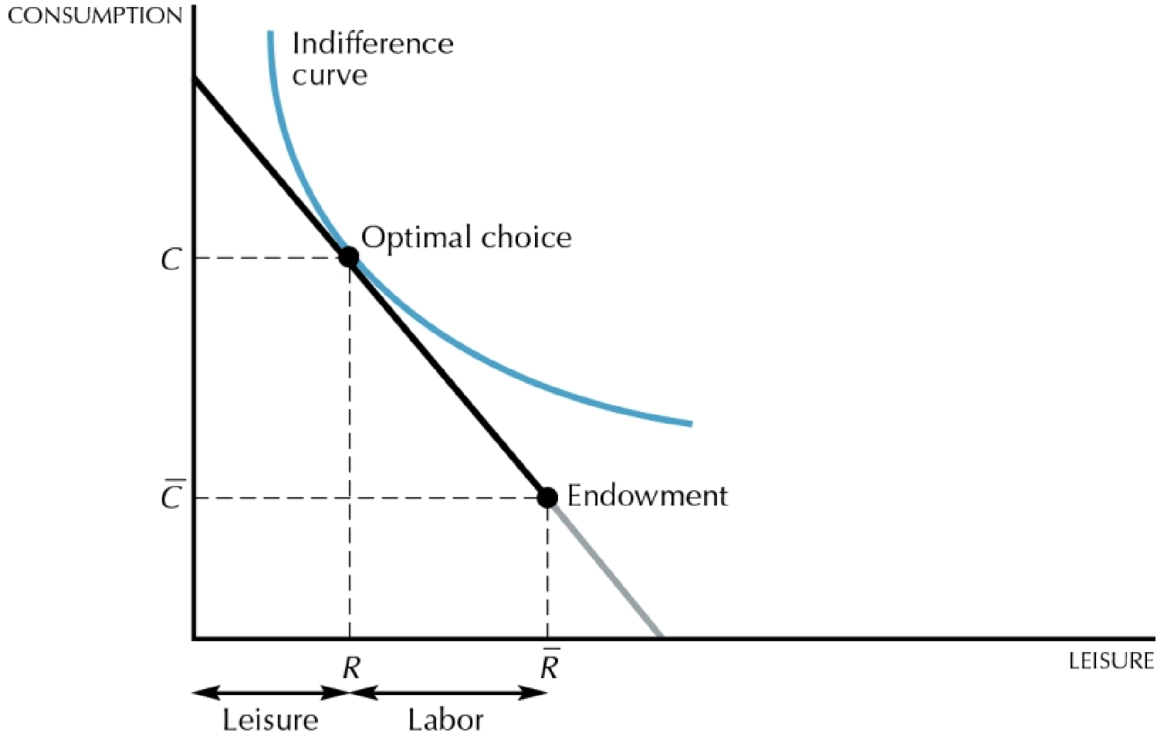
where R is the number of hours this worker allocates for his leisure activity. Then,

$$\begin{aligned} p_C C + wR &= w\overline{R} + m \\ \text{Expenditure} &= \text{Income} \end{aligned}$$

Rearrange to

$$C = -\frac{w}{p_C} R + \frac{w\overline{R} + m}{p_C}$$

Notice that the slope of this budget line is the 'real wage' $-w/p_C$. The optimal allocation between labour and leisure takes place where the worker's indifference curve is tangent to the budget line.



Allocation of time between work and leisure (Figure 9.8 from Varian). The worker works $\overline{R} - R$ hours and allocates R hours for leisure.

When the wage rate increases, does a worker always work more?

Answer: not necessary!.

When the wage rate increases, a worker will work more (due to the substitution effect) but an increase in the wage also increases the value of his endowment $\left(w\overline{R}\right)$ in which this worker can increase his consumption of leisure (endowment income effect). Therefore, the result from a change in wage rate on the labour supply is not clear!

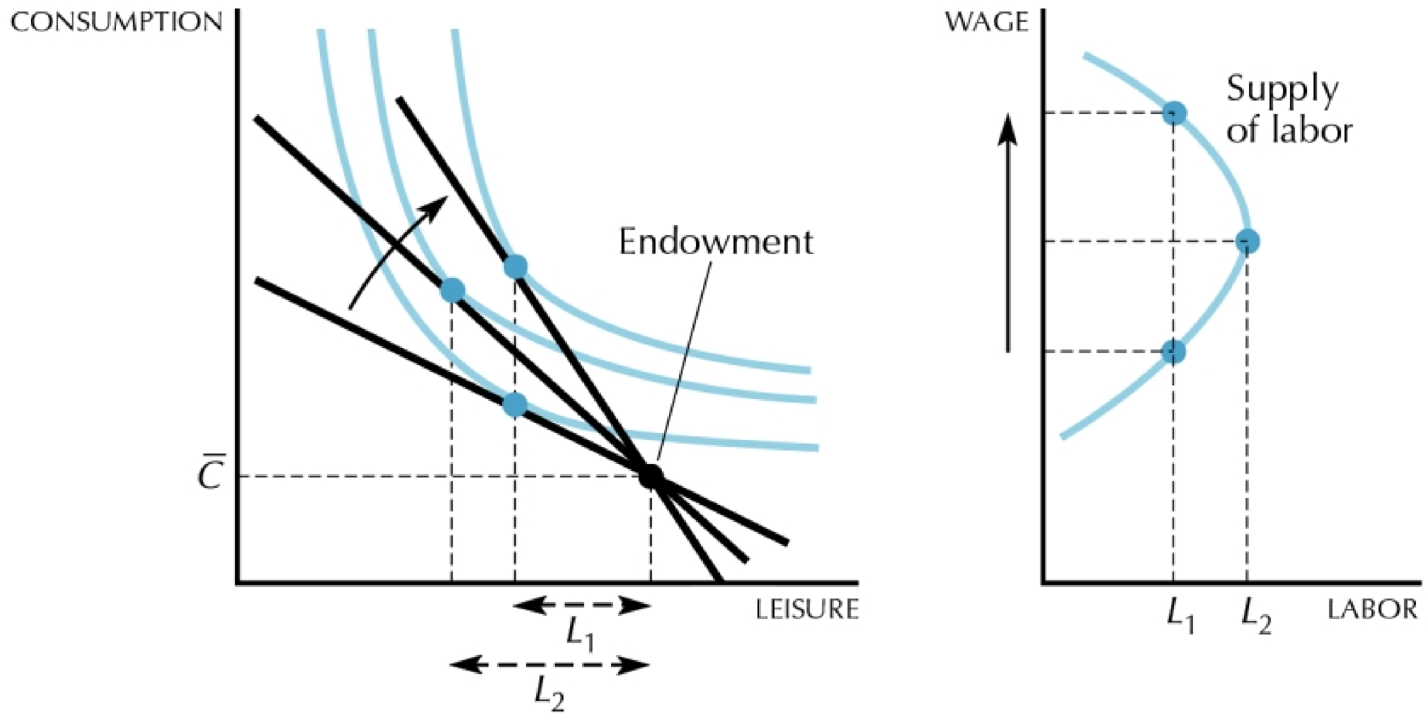
Backward-bending Labour Supply

When $\overline{R} - R = 0$, an increase in w results in a pure substitution effect and an increase in labour supply.

When $\overline{R} - R$ is relatively low, an increase in w results in an increase in labour supply because the substitution effect dominates the income effect.

When $\overline{R} - R$ is relatively high, the result is reverse because the income effect dominates the substitution effect. This happens because the worker uses an additional income from an increase in w to 'purchase' leisure.

These result in a backward-bending supply of labour.



Backward-bending supply of labour (Figure 9.9 in Varian)

7. Reading (for this lecture)

Varian Chapter 8-9

8. The Next Lecture

We will study the consumption allocation between 'today' and 'tomorrow'. A consumer can consume now or save for later. We will investigate the effect of interest rate and inflation on the consumer's decision to consume and save. Also, we will see the role of financial assets and the consumer's decision to invest between risky and riskless assets.

9. Reading (for the next lecture)

Varian Chapter 10, 11, 13; Binger and Hoffman Ch. 18.1-18.4